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MEASUREMENT OF THE TEMPERATURE IN THE UPPER ATMOSPHERE TO 150 km IN A ROCKET EXPERIMENT

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The temperature of the upper atmosphere has been measured previously as high as 90 km by observing the speed of sound at high altitude in experiments in which grenades are released from rockets and exploded.[1-3] It may also be inferred from spectrographic observations of the airglow and the aurora to much greater altitudes.[4] We wish to report briefly the results of the temperature measurements between 100 km and 150 km by an experiment which involves the observation of sunlight resonantly scattered from sodium atoms in a cloud ejected from a rocket. The experiment was performed in the evening twilight of March 2, 1960, at Hammaguir (near Colomb-Bechar), Algeria, and employed a Veronique rocket which rose to 188 km and released 280 grams of sodium. The cloud was visible in the light of the scattered D lines and the altitude of various points accurately determined by triangulation. The temperature was measured by a variation on a technique developed for the airglow by Bricard and Kastler.[5] Photographs of selected portions of the cloud of known altitude were obtained repeatedly from a few moments after release of the sodium until the illumination by the sun was extinguished about half an hour later. These consisted of images formed side by side in light which passed through optical systems which were identical except for the presence of an absorption cell in one. This cell contained sodium vapor at a known temperature T_a and opacity τ. Densitometer measurements of corresponding points in the two photographs were made and the ratio of the intensities recorded at these points was obtained. For a single hyperfine component and an optically thin cloud the intensity measured without the absorption cell is proportional to

$$I_0 = \int \exp(-x^2)dx, \tag{1}$$

where

$$x^2 = Mc^2(\nu-\nu_0)^2/\nu_0^2RT, \tag{2}$$

since the line shape is purely Doppler. Here M is the atomic weight of sodium, T the temperature of the sodium cloud and hence of the atmosphere, R the gas constant per mole, and ν_0 the frequency at the center of the component. The intensity, for the same point in the cloud, measured through the cell is proportional to

$$I_a = \int \exp(-x^2)\,\exp[-\tau\,\exp(-ax^2)]dx, \tag{3}$$

where

$$D = T/T_a. \tag{4}$$

The ratio I_0/I_a is thus a function of T alone if T_a and τ are known and fixed. Complications arise because the cloud is not always optically thin, because the measured ratios are for the sum of all hyperfine components,[6] and because the cloud is in the shadow of the earth's own sodium layer during most of the experiment. This last produces a serious distortion of the spectrum but one which can be allowed for. The altitude and abundance of the earth's sodium were measured and the transmission function computed by meth-

Table I. Summary of results.

Altitude, km	Temperature in °K
100	215 ± 25
110	240 ± 30
120	275 ± 45
130	325^{+65}_{-50}
140	400^{+80}_{-70}
150	515^{+150}_{-115}

ods developed for the twilight flash.[7]

The results for the temperature given in Table I are in agreement with the values deduced by Hunten[4] from a consensus of spectroscopic observations on the airglow and the auroras but far below the values used in the ARDC (Air Research and Development Command) model atmosphere.[8]

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SPECIFIC HEAT OF LIQUID He^3 DOWN TO 0.054°K*

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The intriguing question of whether liquid He^3 may undergo a phase transition into a superfluid state has acquired new interest with the recent predictions of several investigators[1-3] that a transition into such a cooperative state should occur at a sufficiently low temperature. These theories based on the BCS model of superconductivity in metals predict a transition temperature as high as 0.08°K, this temperature depending sensitively on the effective mass m^* of the quasi-particles and on the single-particle potential used in the calculations. No evidence for such a phase transition above 0.03°K was found by Anderson, Hart, and Wheatley[4] in measurements of the coefficient of self-diffusion and nuclear susceptibility. However, in view of the lack of any firm theoretical predictions about the coefficient of self-diffusion near the phase transition, the indication that there exist relative angular momentum states favorable to a transition which would yield no change in susceptibility,[5] and because in the analogous case of superconductors a surprisingly small decrease in the electron spin susceptibility in the superconducting state has been found in superconducting Sn and Hg,[6-8] it would seem desirable to have other evidence before ruling out a phase transition in this region.

In going from the normal to the superfluid state a discontinuous increase in the specific heat of about a factor of two is predicted,[2] thus making the measurement of specific heat a particularly sensitive test for such a transition. Earlier specific heat measurements of Brewer, Daunt, and Sreedhar[9] extending down to 0.085°K showed no anomaly. In the measurements reported here the specific heat of liquid He^3 at saturated vapor pressure has been measured down to 0.054°K and a linear dependence on temperature was found below 0.09°K. A phase transition above 0.054°K would seem, therefore, to be excluded.

The He^3 used in this experiment has been purified by pumping back 2/3 of a given amount

of liquid He^3 at 0.4°K (starting purity, 99.9% He^3). This process was carried out twice giving a He^4 content estimated to be less than 1 part in 10^5. No trace of He^4 was found using a mass spectrometer which could detect a concentration of 0.01% He^4.

The liquid He^3, contained in a copper calorimeter can, was thermally connected to a cerium magnesium nitrate thermometer pill by a large number of No. 40 copper wires. The calorimeter was cooled by demagnetizing a large (150-g) pill of chromic methylammonium alum from a starting temperature of about 0.4°K obtained with a He^3 cryostat. Thermal contact and isolation between the working salt and the copper calorimeter was obtained by means of a superconducting tin heat switch. The specific heat was measured by observing the temperature rise produced by a known heat pulse applied to an electric heater wound on the calorimeter can.

The time of a typical pulse was from 1 to 10 sec, with powers ranging from 300 to 5000 ergs/sec. The largest relaxation time of a pulse was about 20 sec; the heat leak below 0.1°K was about 0.5 erg/sec. Further details of the apparatus and procedure are deferred to a later publication.

We estimate the over-all error in both the specific heat and the temperature to be ±3%. These errors arise almost entirely from a systematic error of ±3% in temperature due to instabilities in the ac bridge during calibration.

The experimental results for several runs and differing amounts of He^3 in the calorimeter are shown in Fig. 1. The open circle points between 0.1 and 0.2°K are given less weight because of the uncertainties arising from a large background heat leak which occurred during these measurements. Below 0.09°K the smooth curve drawn through the experimental points is a straight line through the origin in agreement with the predictions of Landau,[10] and Brueckner and Gammel,[11] who indicate that at a low enough temperature He^3 will behave as a degenerate

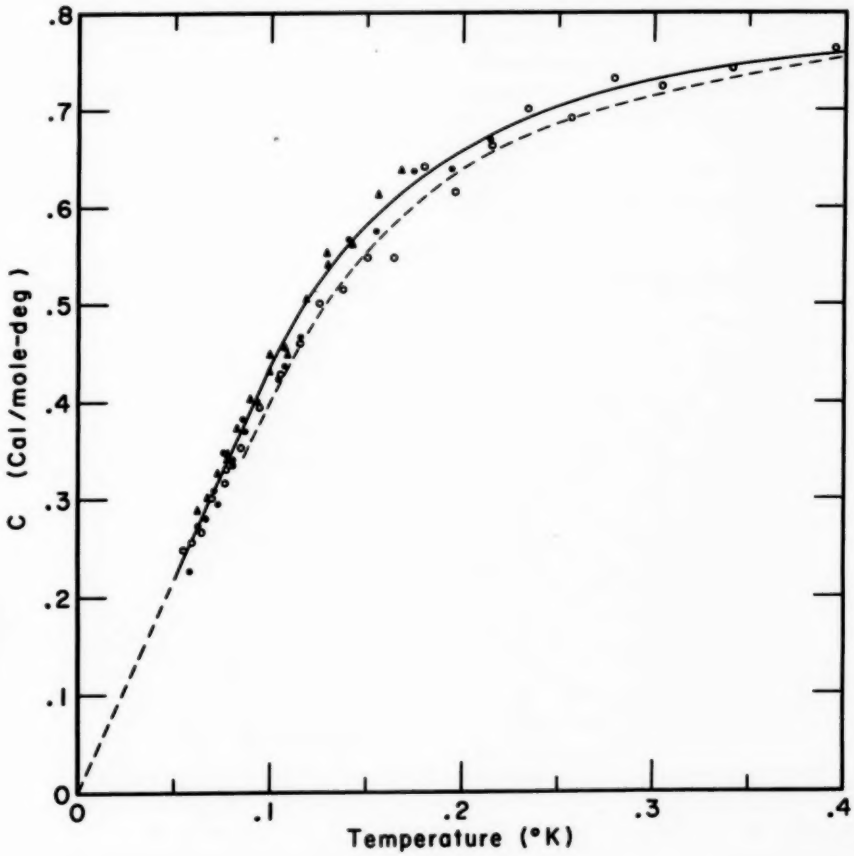

FIG. 1. The specific heat of liquid He^3 at saturated vapor pressure versus temperature. The dashed line is the smoothed curve through the experimental points of Brewer et al.[9] The open circles, closed circles, and triangles represent data obtained with 0.0319 mole, 0.0306 mole, and 0.0207 mole of He^3, respectively, in the calorimeter.

Fermi liquid with constant effective mass (see also Goldstein[12]). We obtain a limiting specific heat of $4.38\,T$ which is somewhat higher than the value of $4.00\,T$ quoted by Brewer et al.[9] For a Fermi liquid near 0°K the effective mass m^* and the measured specific heat C are related to C_F, the specific heat of an ideal Fermi gas of mass m, by $C/C_F = m^*/m$. m^*/m obtained from our data is 2.19 ± 0.13.

Table I gives the values of entropy calculated from our specific heat data. The entropy at 0.23°K is 0.92 ± 0.05 cal/mole-deg, which can be compared with the value of 0.96 ± 0.03 cal/mole-deg given by Weinstock, Abraham, and Osborne[13] and the value of 0.86 cal/mole-deg taken from the data of Brewer et al.[9] The latter quote an error of ± 0.01 cal/mole-deg and a possible systematic error of $+0.03$ cal/mole-deg due to the extrapolation of their data below 0.1°K.

We would like to thank Professor J. G. Daunt for suggestions in the design of our ac bridge, J. D. Reppy and Dr. D. J. Sandiford for many helpful suggestions, and D. Johnson for his expert help in the construction of the apparatus.

Table I. Entropy of liquid He^3 at saturated vapor pressure.

T (°K)	S (cal/mole-deg)
0 to 0.09	$4.38T$
0.10	0.4378
0.12	0.524
0.14	0.606
0.16	0.683
0.18	0.756
0.20	0.824
0.23	0.918

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MAPPING OF THE FERMI SURFACE BY A COMBINATION OF GEOMETRIC RESONANCE AND TILT EFFECT

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Recent experiments[1,2] have indicated an increase in the attenuation of sound waves in semimetals when there is a magnetic field which is tilted from a direction transverse to the direction of propagation. Theoretical work has confirmed that in the high-field limit,[3] the increase in the ultrasonic attenuation arises when the carriers drifting along the magnetic field have a component in the direction of propagation that is equal to the sound velocity V_S. The carrier is then in phase with the sound wave and can experience a steady acceleration, effectively absorbing energy from the wave. The onset of the increase in attenuation occurs at a critical angle ν_c given by $\sin\nu_c = V_S/V_F$, where V_F is the Fermi velocity of the fastest group of carriers in the direction of the magnetic field. The high-field tilt effect, therefore, offers the possibility of obtaining the Fermi velocity point by point on the Fermi surface.

Other experiments[1,4,5] have indicated that there are magnetoacoustical oscillations in the attenuation as a function of magnetic field in a transverse magnetic field. The theoretical explanation[6] indicates that maxima and minima in the oscillations occur when the cyclotron orbit diameter is equal to an integral or half-integral number of wavelengths, respectively. From the periods of the oscillations with magnetic field, the extremal dimensions of the Fermi surface normal to the magnetic field can be determined.

The interesting possibility of combining the tilt effect and geometric resonance experiments to obtain information about the dimensions of nonextremal orbits on the Fermi surface now presents itself. The main contribution to the attenuation, when the magnetic field is tilted towards the direction of propagation by angles larger than ν_c, comes from those particular orbits that are drifting along the field in phase with the sound wave, provided that $\omega\tau$ is large enough. By varying the angle of tilt, one can vary the orbit or orbits which dominate the attenuation. If one now simultaneously performs a geometric resonance and a tilt effect experiment, one can obtain from the period of the geometric resonances the dimensions of these nonextremal orbits as well as their drift velocity along the magnetic field. The physical situation as described above is pictured in Fig. 1(a) and (b). The drift velocity has a component along the direction of propagation equal to V_S so that its

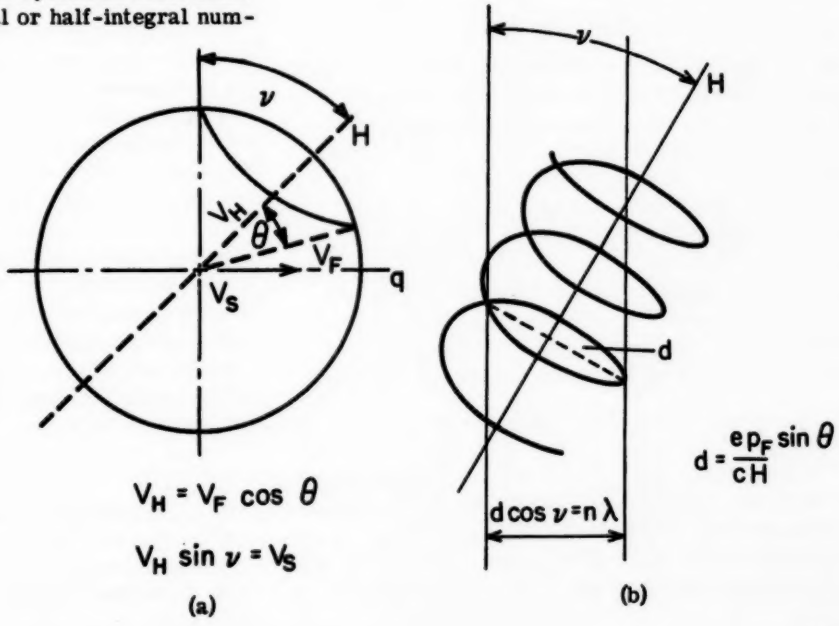

FIG. 1. (a) The orbit which dominates the attenuation when the magnetic field is tilted in the direction of the sound wave is shown to drift along the magnetic field with a velocity $V_S/\sin\nu$. (b) When the field is tilted the orbit gives rise to a maximum in the attenuation when the component of the orbit diameter in the direction of propagation is equal to $n\lambda$.

magnitude along H can be obtained from the angle of tilt, $V_H = V_S/\sin\nu$.

The conditions for this phenomenon to be observable are (1) that there exist sections of the Fermi surface with a Fermi velocity small enough so that the effect occurs at measurable large angles of tilt, and (2) that $\omega\tau$ be ≥ 10 so that the contribution of the orbit drifting in phase with the sound wave dominates those from all other orbits. The first requirement would seem to be satisfied by the semimetals and by certain portions of the Fermi surfaces of Sn, Zn, Mg, Ga, etc. The second condition would require materials of ultrahigh purity together with microwave sound waves except perhaps for the semimetals, Sn, Zn, and maybe others.

Detailed calculations have shown that the geometric resonances for tilted magnetic fields arise from the field dependence of the conductivity tensor. The calculations have been made using spherical Fermi surfaces for the sake of simplicity. In the case of longitudinally polarized waves, the σ_{zz} component of the conductivity tensor plays the essential role. In the region of the geometric resonance (i.e., $X = qV_F/\omega_C \geq 1$) and for large $\omega\tau$ we find

$$\sigma_{zz} = [3\sigma_0/(ql\sin\nu)^2] \times (1 - i\omega\tau)J_0^2(X\cos\nu\sin\theta^*)[N - iM],$$

where N and M are functions[3] of $\omega\tau$ and ν but not of magnetic field, and

$$\theta^* = \cos^{-1}(V_S/V_F\sin\nu).$$

Also, q is the sound wave number and l is the mean free path of the carriers. The Bessel function gives rise to the oscillation in the attenuation with the magnetic field. In evaluating the expression for σ_{zz} we have used the presence of denominators in the integrals involved which give a resonance whenever $\cos\theta = V_S/V_F\sin\nu$ for large values of $\omega\tau$. This is the condition for the steady acceleration of carriers travelling on an orbit at an angle θ to the magnetic field. From the period of the oscillations, the dimensions of the Fermi surface can be determined by using

$$q\cos\nu V_F\sin\theta^*\Delta(1/\omega_C) \approx \pi.$$

The attenuation has been calculated using several of the models developed by others.[6,7] Among these models are the free-electron model, the two-band model with deformation potential for semimetals, and the case of minority and majority carriers with a positive background. All the cases considered show geometric resonance in a tilted field in a fairly similar way. In Fig. 2 we have plotted a normalized attenuation versus magnetic field for the case of the minority and majority carriers. The oscillations of the attenuation with the magnetic field are very much stronger than for transverse fields, to which all the orbits contribute. The theoretical calculations thus support in detail the physical arguments given about obtaining the dimensions of nonextremal orbits on the Fermi surface through a combined geometric-resonance tilt-effect experiment.

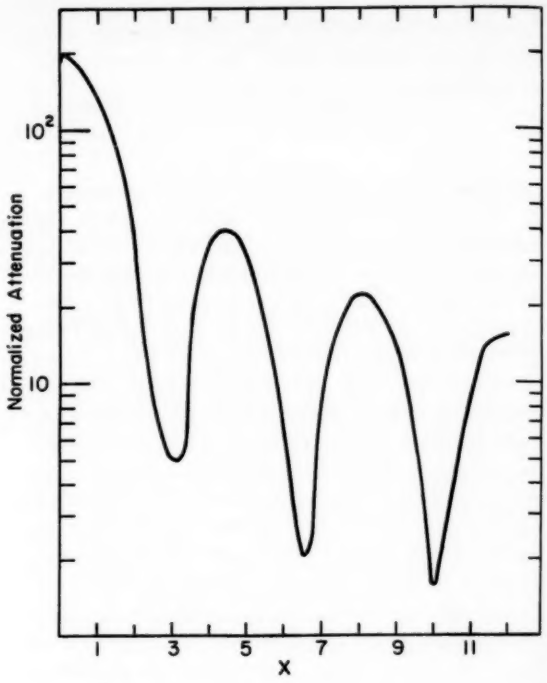

FIG. 2. The normalized attenuation is plotted versus $X = qV_F/\omega_C$ for a model consisting of minority and majority carriers with $V_F(\text{maj})/V_F(\text{min}) = 10$. The angle of tilt in this case is $\nu = 0.02$ and $\omega\tau = 10$. The plot is correct for all magnetic fields when the screening of the transverse currents breaks down and is correct for $X > 2$ when screening does not break down.

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CYCLOTRON RESONANCE IN INDIUM

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Cyclotron resonance absorption in indium reveals carriers having several values of effective mass. The results of preliminary measurements at 9.1 kMc/sec are described here, for one orientation.

A large single crystal was grown from Tadanac special research grade indium by means of the modified Bridgman technique. The experimental samples were cut, by means of a spark cutter,[1] in the shape of thin rectangular slabs, measuring approximately $2\times10\times20$ mm. Each slab was electropolished after cutting and its surface served as about 10% of the surface of a microwave cavity in the reflection spectrometer previously described.[2] The cavity was maintained reasonably close to match ($\Gamma\sim0.1$) with changes in reflection, $\Delta\Gamma$, corresponding to absorption. A 60-cps modulation of amplitude H_M was superposed on the steady field H_0. The output of the synchronous detector was presented on an X-Y recorder with X proportional to H_0 and Y to the 60-cps component of dR/dH.

Absorption signals were observed when the temperature was 2.1°K or below. Figure 1 is an example which shows well-defined peaks, together with some weak ones. This recording was obtained at 1.54°K with H_0 along the [111] axis and $H_M\sim20$ oersteds. The microwave magnetic field was parallel to H_0 and approximately parallel to the length of the sample. The large peak below 200 oe was found to be associated with the superconducting-normal transition. Repeated runs up to as high as 8000 oersteds gave reproducible series of maxima, from which the effective masses shown in the first column of Table I were calculated. In addition, there appear a number of other peaks up to 7000 oersteds to which no unambiguous assignment of masses has been made.

Indium is trivalent, and has a face-centered tetragonal structure only slightly distorted from the fcc structure of aluminum. Harrison[3] has shown that the free-electron model is reasonably successful in interpreting the cyclotron resonance data for alumium. One would therefore expect the model to be a fair approximation for indium too. The portion of the Fermi surface in the second zone in indium will be tetragonally

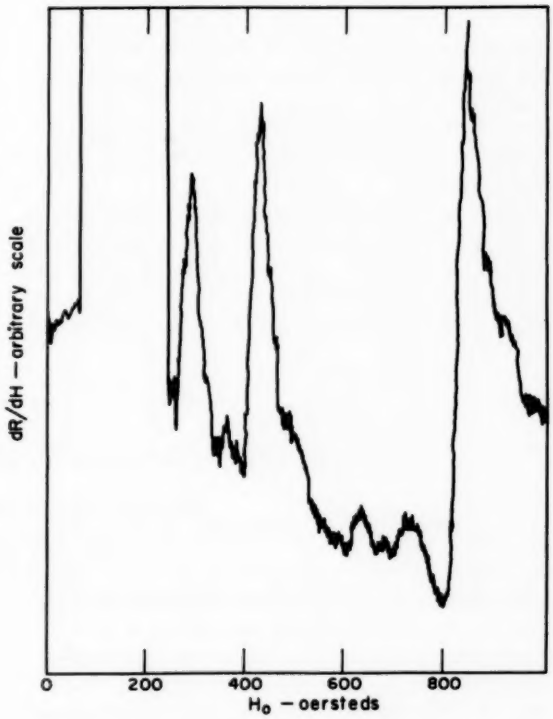

FIG. 1. Cyclotron resonance absorption in indium. The ordinate is linear with the 60-cps absorption component of $\Delta\Gamma$ of the sample cavity. Conditions: $T=1.54$°K, H_0 parallel to the [111] crystal axis, $f=9.1$ kMc/sec, and $H_M\sim20$ oe.

Table I. Cyclotron resonance masses for H_0 parallel to [111].

Experimental	Theoretical[a] Value	Orbit
0.222 ±0.006	0.23	Type *B* arms in 3rd zone
0.255 ±0.005	0.29	Type *A* arms in 3rd zone
0.79 ±0.04		
0.88 ±0.04		
0.94 ±0.04	0.94	$k_H \sim 0$ orbits in 2nd zone
1.3 ±0.1		

[a]From the free-electron model.

distorted, and the arms in the third zone will fall into two types: those with their axes along ⟨110⟩ directions, and the rest with their axes along ⟨101⟩ directions. These two will henceforth be referred to as type *A* and type *B* arms, respectively.

The theoretical effective masses shown in Table I were obtained graphically by the method described by Harrison.[4] The mass of 0.29 arises from elongated orbits on type *A* arms, and 0.23 from similar orbits on type *B* arms. The kind of distortion of these arms that has been suggested, in the case of aluminum,[5] would, when applied to indium, increase the number of electrons contributing to the above two masses. It may be pointed out here that Shoenberg[6] obtained a mass of 0.30 from de Haas-van Alphen measurements with the magnetic field along the [110] direction. The mass of 0.94 arises from orbits in the second zone which have a vanishing or very small component of momentum in the field direction.

The study is being extended to samples of other orientations and to higher frequencies. Helpful discussions with D. W. Feldman are gratefully acknowledged.

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INFLUENCE OF DISLOCATION ON DIFFUSION RATE OF *F* CENTERS IN KCl

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In a previous paper[1] it was suggested that at temperatures below 500°C the diffusion of *F* centers in a KCl crystal might take place predominantly along disordered interfaces. In the present report an experimental confirmation of this "diffusion along disordered interfaces" will be given.

The basic idea underlying our method of establishing such a predominance and estimating its rate is the following. If in a crystal an abnormally high density of dislocations along a definite orientation is created, e.g., by microscopic deformation, then the crystal should show a distinct anisotropy of *F*-center diffusion after additive coloring.

In the experiments KCl was purified by crystallizing it twice by the Kyropoulos method. From the single crystal thus obtained rectangular blocks were cut of $1.0 \times 1.5 \times 2.0$ cm^3. These blocks were deformed in an elastic clamp. After 60 minutes exposure to the pressure the rate of compression along [100] was 7.3%. The deformation of the crystals resulted in only a single pair of mutually perpendicular slip systems as indicated in Fig. 1.

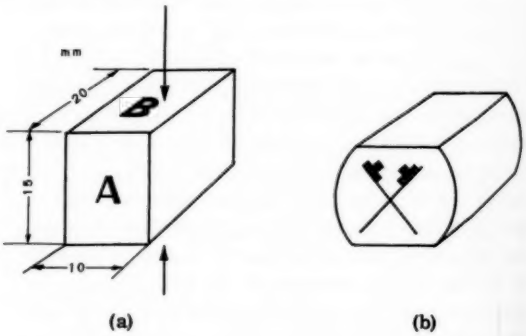

FIG. 1. A crystal employed for the experiment. (a) Before compression. (b) After 7.3% compression, dislocations belonging to one single slip system only are present.

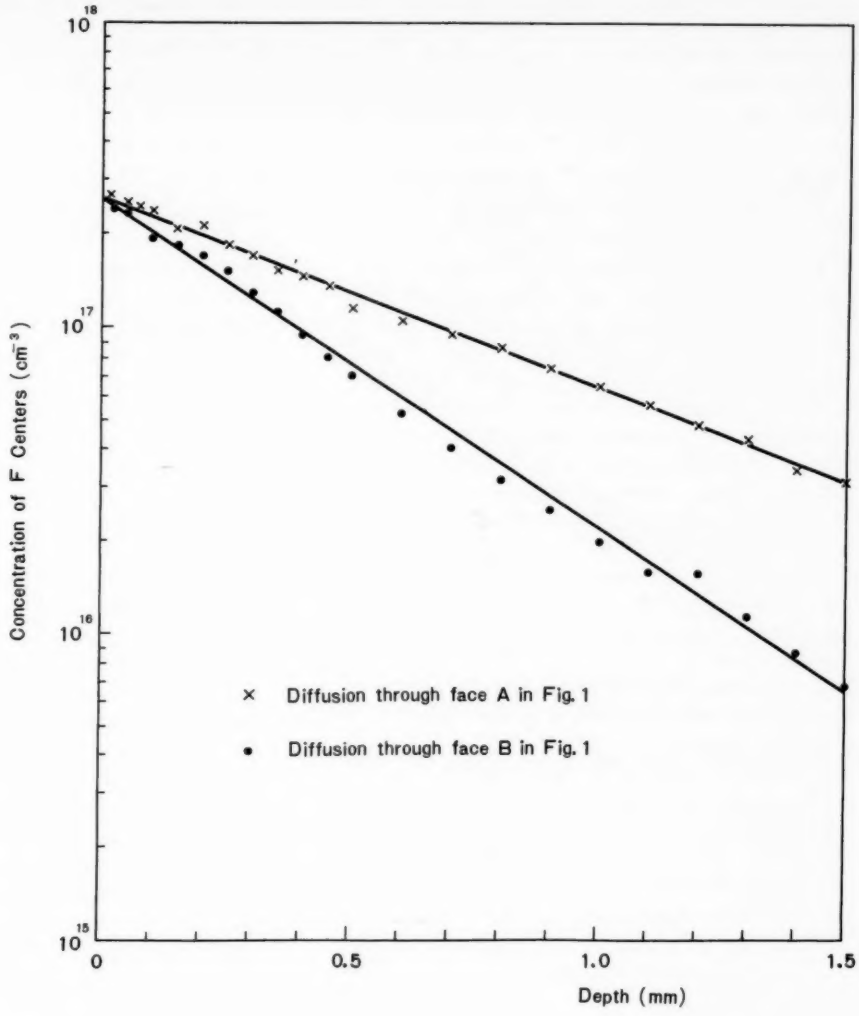

FIG. 2. Penetration curves of F centers in KCl through different lateral faces.

The deformed crystal was colored by heating it to 450°C in potassium vapor. A heating time of 20 hours was found to be sufficient for obtaining colored crystals which showed a marked concentration gradient of F centers. The concentrations in different regions of the crystal were measured optically, using a thin beam of monochromatic light.

The measurements were carried out in the center of each exposed face in order to avoid interferences from F centers which had penetrated through the other faces. The results are shown in Fig. 2. It appears that the rate of F-center diffusion through face A, that is along the orientation of produced dislocations, is nearly three times as fast as that through face B. It seems reasonable to conclude that this anisotropy is due to the pronounced orientation of the crystal dislocations. Assuming a dislocation density of 10^9 cm^{-2} for 7.3% deformation, a dislocation width of 10^{-7} cm, and a volume diffusion coefficient of F centers of 3.9×10^{-9} cm^2 sec^{-1} at 450°C,[1] a diffusion coefficient along a disordered interface (dislocation) can be derived according to Fisher's formula.[2] This is found to be 3.7×10^{-8} cm^2 sec^{-1} at 450°C.

The authors wish to express their sincere thanks to Professor H. Kawamura for suggesting the experiment.

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SIGN OF KNIGHT SHIFT IN SAMARIUM INTERMETALLIC COMPOUNDS

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Measurements of the Al^{27} nuclear resonance in rare-earth aluminum intermetallic compounds have shown that in these compounds the Knight shifts are large, temperature dependent, and may have either sign, depending on whether the spin polarization of the rare earth ions is parallel or antiparallel to the applied field.[1] These observations in general accord with the Zener-Ruderman-Kittel-Yosida mechanism[2] whereby conduction electrons are polarized by an exchange interaction between them and the rare earth ion spins, provided one assumes that this interaction has the opposite sign from that originally proposed by Zener. The samarium compound is unlike the others, however, in that the sign of the rare earth contribution to the Knight shift reverses in going from 77°K to room temperature.[3] It is the purpose of this note to show that this anomalous reversal in $SmAl_2$ is a consequence of theory when the second-order Zeeman effect is included.

We shall at first ignore the effect of the crystalline field, and assume that the exchange fields which the rare earth ions exert on each other, and on the conduction electrons which they polarize, are proportional to the expectation value of the spin of the samarium ion. If we neglect saturation, the formulas for the magnetic moment of the Sm^{+++} and for this expectation value are

$$M_{Sm} = aN\beta(H_0 - 5H_{ex})T^{-1} + bN\beta(H_0 + 2H_{ex}), \quad (1)$$

$$\langle S_z\rangle_{Sm} = \tfrac{5}{2}a(H_0 - 5H_{ex})T^{-1} - b(H_0 + 2H_{ex}), \quad (2)$$

where

$$a = \beta g^2 J(J+1)/3k = 5\beta/21k, \quad b = 20\beta/7h\nu_{7/2,5/2}.$$

Note especially that the coefficients of H_0 and H_{ex} have the opposite sign in the $1/T$ term; this is because in Sm^{+++} $g = 2/7$, and so $2(g-1)/g = -5$. The factor 2 in the second part of (1) or (2) has its origin in the fact that the matrix elements of $2S$ nondiagonal in J are double those of $L+2S$ since $L+S$ is diagonal. The above formulas are obtained by adapting the standard expression[4] for the susceptibility of Sm^{+++} to include an exchange field H_{ex} acting only on the spin in addition to the applied field H_0, in essentially the same fashion as was done by Wolf and Van Vleck for europium garnet.[5] In the case of Sm^{+++}, it is sufficient for our purposes to consider only the lowest multiplet component as inhabited, but it is essential to include the part of the susceptibility arising from the "temperature-independent paramagnetism," or second-order Zeeman effect, represented by the second part of (1) or (2).

The "crossover point," i.e., point of change in sign for the contribution of the Sm^{+++} to the Knight shift, is that at which both $\langle S_z\rangle = 0$ and $H_{ex} = 0$. It is consequently given by $T_{CO} = 5a/2b$. [The exchange field which polarizes the conduction electrons is not necessarily the same as that in (1) or (2) acting on the rare earth ions, but both are taken proportional to $\langle S_z\rangle$.] The interval $h\nu_{7/2,5/2}/k$ is about 1500°K, and so according to (2) the value of T_{CO} is about 300°K. The predicted sign behavior is in accord with experiment; i.e., the Sm^{+++} contribution to the Knight shift in $SmAl_2$ has at low and high temperatures, respectively, the opposite and the same sign as the Gd^{+++} contribution to the Knight shift in $GdAl_2$. The observed value of T_{CO} is about 150°K. However, T_{CO} is hard to locate with precision, and also the above theory is a crude one which neglects such refinements as the crystalline field, and the effect of orbital orientation on exchange coupling.

Since the Sm^{+++} ion is known to be at a site of cubic symmetry,[6] the effect of the crystalline field is to split the $J = 5/2$ level into a doublet and a quartet. Formulas (1) and (2) must then be modified by multiplying the $1/T$ terms by the factor[7]

$$f(T) = \frac{5/21 + (26/21)e^{-h\nu_c/kT} + (32\,kT/21h\nu_c)\left(1 - e^{-h\nu_c/kT}\right)}{1 + 2e^{-h\nu_c/kT}}, \quad (3)$$

where $h\nu_c$ is the splitting between the doublet and the quartet, and has a sign which is positive if the doublet lies lowest and negative if the quartet lies lowest. The temperature-independent terms in (1) and (2) will be little affected if crystalline splittings are small compared to the multiplet separation. It is seen from (3) that the effect of a crystalline field will, in general, be to reduce T_{CO}, thus giving improved agreement with experiment. The reduction will not be great, however, unless $h\nu_c \sim kT_{CO}$. Further discussion of this point must await a more accurate experimental value for T_{CO}.

At the crossover point, the susceptibility of the Sm^{+++} ion in an applied field should not be influenced by the exchange couplings between Sm^{+++} ions. This accords with measurements of Williams and Sherwood who find that at room temperature the susceptibility is[8] $\chi_M = 9.9 \times 10^{-4}$, in agreement with the theoretical value for free, uncoupled Sm^{+++} ions.[4]

In closing we may note an interesting reciprocal relation between (1) and (2). The temperature at which an applied field generates no $\langle S_z \rangle$ is also the temperature at which an exchange field, acting alone, produces no magnetic moment. Hence in, say, samarium garnet, the exchange field from the ferric ion should induce no magnetic moment in the samarium in the vicinity of room temperatures. So it is experimentally. In fact, the difference between the magnetic moments of YIG and SmIG is found by Pauthenet to be practically zero throughout the range from 0°K to the Néel point.[9] The vanishing of the susceptibility in an exchange field at a particular temperature is not by itself enough to explain the magnetic inertness of the samarium in the garnet for all temperatures, and presumably at low temperatures the effect of the crystalline field should be included. Our attempts to obtain theoretically the requisite inertness have not yet met with quantitative success, and more complete experimental data on the magnetic behavior of the samarium garnets are highly desirable.

We are indebted to V. Jaccarino and H. J. Williams for informing us of experimental results prior to publication and for valuable discussions. One of the authors (J. W.) wishes to acknowledge the hospitality of the Bell Telephone Laboratories during the summer of 1960 where part of this research was carried out.

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FORMATION OF ALPHA CLUSTERS IN EVEN-EVEN NUCLEI

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In the alpha decay of even-even nuclei, the transition probabilities to the 0+ ground states and the 2+ first excited states are of the same order of magnitude, while those to the 4+ and higher states are considerably smaller. In addition, the variation in the hindrance factors for decay to a given level is large. Several attempts[1] were made to explain this behavior in terms of the initial distribution of the alpha clusters in the nuclear surface. The purpose of this Letter is to interpret this fact from a different point of view, considering the formation probability of alpha clusters of different angular momenta.

In view of the fact that a pairing energy exists in nuclei with mass number less than about 120 and is proportional to $2j+1$ for pairs in a j shell, it is reasonable to assume, for a deformed nucleus with axial symmetry, a pairing energy between a pair of nucleons of opposite Ω (the quantum number for the projection of the total particle angular momentum along the symmetry axis) proportional to

$$\sum_j (2j+1)|C_{j\Omega}|^2,$$

where $C_{j\Omega}$ are the coefficients in the expansion of the particle wave function in terms of the eigenfunctions of the total particle angular momentum $\vec{j}^2$. Since $j > |\Omega|$, for states with large Ω, only eigenfunctions of $\vec{j}$ with relatively large j values are present in the expansion, while for states with small Ω, components with small j values may appear. Therefore, the pairing energy for a pair of high-Ω nucleons is expected to be generally larger than that for a pair of low-Ω nucleons.

Similarly, the wave function for an alpha cluster in an axially symmetric deformed oscillator potential can be written

$$\psi_{\Omega'} = \sum_l C_{l\Omega'} \psi_{l\Omega'},$$

where l is the orbital angular momentum quantum number of the cluster, and Ω' is the projection quantum number for l on the symmetry axis. The relative abundances of different angular momentum groups of clusters in the nucleus are determined by the quantities $|C_{l\Omega'}|^2$.

It is seen that the formation of high-angular-momentum groups is hindered for two reasons:
(a) The formation of clusters of high Ω' needs the breakup of pairs of nucleons of high Ω.
(b) In order that a cluster wave function of low Ω' may contain components of large l, it must be one of sufficiently high-energy states. Hence, the formation of high-angular-momentum groups is energetically unfavored; only nucleons of sufficiently high-energy and low-Ω states can contribute appreciably to the abundance of high-l alpha groups. The degree of hindrance would depend on the distribution of the loose nucleons among the available states in the nuclear surface. In general, if the assumed pairing energy does exist, we may expect that clusters of lower angular momentum are more probable in a nucleus.

On the basis of his results on (p,α) reactions in heavy nuclei, Hodgson[3] has tentatively concluded that the alpha particles knocked out of the target were in S orbits. Further experiments of this kind could be valuable in understanding the process of alpha-cluster formation in nuclei.

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HELICITY OF μ^- MESONS FROM π-MESON DECAY

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The current theory of weak interactions with a single two-component neutrino and lepton conservation predicts a positive helicity for negative μ mesons observed in the rest frame of the parent π meson.[1] Recent measurements by Alikhanov et al.[2] are consistent with this prediction. We report here a measurement of the helicity of negative μ mesons arising from the decay of π mesons produced by the CERN proton synchrotron.

The method used in this experiment was to measure the spin dependence of the differential cross section for the scattering of polarized μ mesons on polarized electrons, by reversing the magnetization in iron sheets to change the spin orientation of the electrons. We measured the energy of the knock-on electrons. This theoretical cross section is given[3,4] by

$$\sigma(E,\vec{P}_e\cdot\vec{P}_\mu)dE=\frac{2\pi r_0^2 m_0 c^2}{\beta_\mu^2 E^2}\left\{1-\beta_\mu^2\left(\frac{E}{E_m}\right)+\tfrac{1}{2}\left(\frac{E}{E_\mu}\right)^2 -\vec{P}_e\cdot\vec{P}_\mu\frac{E}{E_\mu}\left(1-\frac{E}{E_m}+\frac{E}{2E_\mu}\right)\right\}dE, \quad (1)$$

where E is the kinetic energy of the struck electron, E_m the maximum possible energy transfer, E_μ the total energy of the incident μ meson, and $\vec{P}$ the polarization.

The polarized μ-meson beam was obtained with the arrangement shown in Fig. 1. π mesons from the target T, in the momentum band 12.1 ± 0.3 Gev/c, were focused by the synchrotron magnets A_1A_2 onto the 2-cm wide slot S in an 80-cm long lead collimator. This beam of some 8×10^5 negative particles per pulse was then collimated by a pair of quadrupoles Q_1Q_2. After a decay path of some 45 m, the 11-Gev/c μ mesons (emitted forward by the π mesons) were focused by a second pair of quadrupoles Q_3Q_4 through a 760-cm graphite filter F. The μ mesons lost about 3 Gev in traversing the filter. A magnet M before the filter dispersed the beam so that with a pair of 7×7-cm^2 counters C_1C_2 it was possible to define a μ-meson beam of 8 Gev/c. The beam had about 15% momentum spread, an intensity of 2×10^3 particles per pulse, and a calculated longitudinal polarization of $|\vec{P}_\mu|=0.85$. The flux of π mesons counted by C_1C_2 was measured to be about 3×10^{-5} of the μ-meson flux. The number of μ mesons in this beam from the decay of K mesons was estimated to be negligible.

To produce, and measure the energy of, the knock-on electrons we used a total absorption shower detector D which consisted of 20 interleaved layers of iron and plastic scintillator sheets. The sheet dimensions were $1\times30\times40$ cm^3, and they were inclined at an angle of 30° with respect to the beam axis. The induction in the iron was $\pm2\times10^4$ gauss. The energy dependence of the detector's output signal was measured with incident electrons of known energy; it was essentially linear in the range 1 to 10 Gev. The resolution, $\Delta E/E$, changed from 50% to 10% in this range.

The experiment was designed to minimize systematic instrumental asymmetries from such causes as change of gain with magnetic field. Measurements taken throughout the experiment showed that such asymmetries were significantly smaller than our statistical errors.

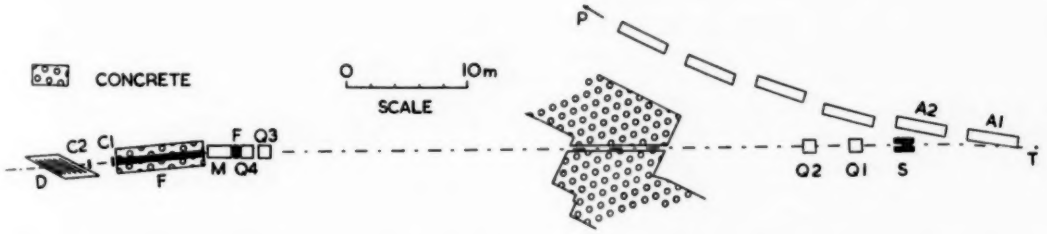

FIG. 1. Experimental arrangement. P, circulating proton beam; A, synchrotron magnets; S, Pb collimator; Q, quadrupoles; M, dispersing magnet; F, graphite filter embedded in concrete; C, counters; D, detector.

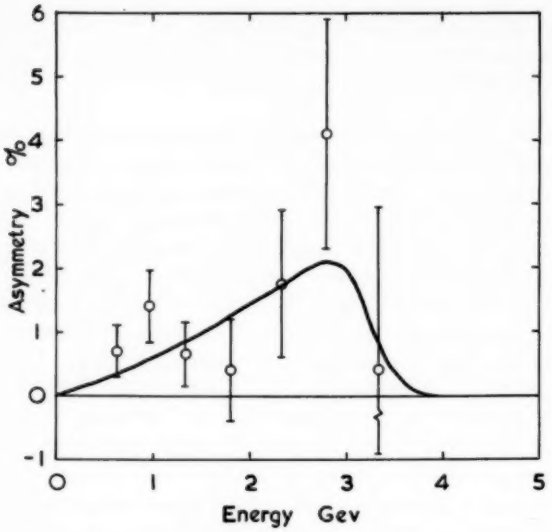

FIG 2. Asymmetry of shower production cross section, $(N_a-N_p)/(N_a+N_p)$, with magnetic field reversal vs shower energy.

The energy spectrum of μ-meson-produced showers above 0.5 Gev was recorded with a pulse-height analyzer. It showed the features predicted by Eq. (1) with a contribution of high-energy showers due to μ-meson bremsstrahlung. At the maximum energy of the knock-on electrons, bremsstrahlung contributes about 30% to the total cross section. These results will be reported elsewhere.

Figure 2 shows the measured difference in shower production cross sections (knock-on electrons and bremsstrahlung) with reversal of the magnetic field as a function of the shower energy. About 1.2×10^7 μ mesons were recorded in the 12.5 hours of normal machine operation during which the data shown in Fig. 2 were collected. The solid curve is the calculated difference for positive helicity. From these data and another run with a beam of 62% polarized μ mesons, the helicity is determined to be

$$\mathcal{H}(\mu) = +1.17 \pm 0.32 \text{ (standard deviation)},$$

which is in agreement with the theoretical prediction of +1.

The authors wish to express their indebtedness to the staff of the CERN Proton Synchrotron Group who made it possible to have the excellent π-meson beam, and to Professor G. Bernardini whose suggestion initiated it.

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MEASUREMENT OF THE RATE $\mu^- + C^{12} \rightarrow B^{12} + \nu$*

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The study of the μ^- absorption process by observation of transitions to definite final states has thus far been carried out only for the reaction $\mu^- + C^{12} \rightarrow B^{12} + \nu$.[1-5] Interest in such transitions centers on the possibility of accurate measurement of the muon capture coupling constants and of detection of the conserved vector current and induced pseudoscalar contributions. However, the experimental results on this reaction are not in agreement with each other, making precise comparison with theory difficult. We have therefore measured this reaction rate again with technique differing from that of reference 5 in several important respects, and with superior accuracy.

The counter arrangement is shown in Fig. 1. A 45-Mev negative muon beam was stopped in a 4×4×2 in. plastic (CH) scintillant. The stopping

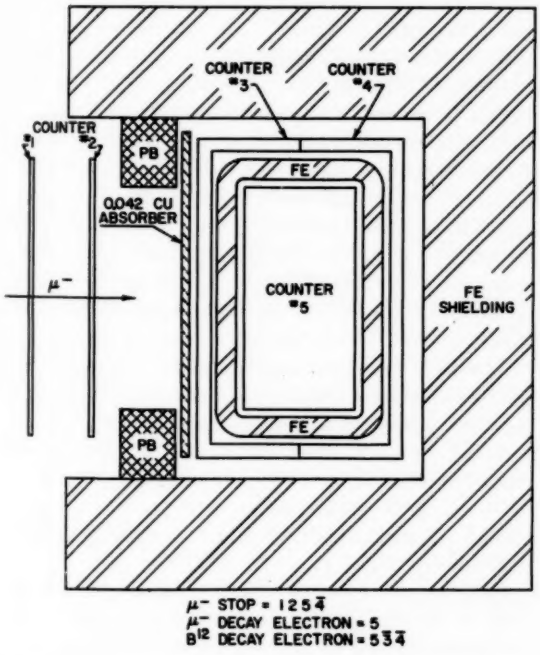

FIG. 1. Apparatus for measurement of reaction rate $\mu^- + C^{12} \rightarrow B^{12} + \nu$. All counters are plastic scintillant. Numbers 1 and 2 are 5×5×1/16 in., No. 5 is 4×4×2 in., and Nos. 3 and 4 are 1/4 in. thick and are shaped to surround No. 5 as completely as possible.

particles included about 1/3% pions.[6] The number of muons decaying in the scintillant was measured by observation in counter No. 5 of their decay electrons with mean life 2.04 μsec. The nuclear absorption to form bound B^{12} was observed (also in counter No. 5) by means of the β-decay electrons from the ground state of B^{12} ($T_{1/2} = 20.3$ msec). The 0.375-in. Fe absorber served to prevent detection of B^{12}-decay electrons by the cosmic-ray shield $\bar{3}\bar{4}$, which was in operation only during the B^{12} measurement. The shielding consisted of the Fe bricks shown, plus a one-foot thickness of paraffin and a massive blockhouse.

The efficiency for detection of μ^--decay electrons in counter No. 5 was high, since the threshold for electron detection could be set at 0.35 Mev without encountering excessive background. The fraction of the B^{12}-decay spectrum which was detected in counter No. 5 was fixed by means of a fast-slow coincidence technique in which the slow pulse provided energy resolution, and the fast pulse provided background reduction by anticoincidence with the cosmic-ray shield counters Nos. 3 and 4. The slow discriminator threshold was frequently calibrated against known γ-ray spectra in counter No. 5. It was thus possible to compute the detection inefficiency due to loss of electrons through the surfaces of the counter, and the loss of a portion of the spectrum due to the finite energy threshold. Confirmation of the accuracy of these computed corrections came from varying the energy threshold, and also the area of the entering beam from 3 sq in. to 16 sq in. At full aperture the beam intensity was also varied to check proportionality to the muon flux. Consistent results were obtained for all measurements.

The experiment consists basically of the measurement of two decay curves, one the μ-e disintegration, the other the B^{12} β decay. The ratio of the total numbers of decay particles observed in the two curves, combined with the muon decay rate in carbon, yields the reaction rate desired. Analysis of muon decay curves is straightforward, since muons stop randomly during a cyclotron pulse (200 μsec long), and their electron decay curve is superimposed on a uniform background. Analysis of our data yielded a μ^- mean life in carbon of 2.05 ± 0.04 μsec, in

good agreement with published data.[7] The B^{12} β decay presents different problems, because zero time is measured from a particular cyclotron FM pulse, and the β decay is measured during a "machine off" period. It is possible that activities other than the desired one are also decaying during the measurement, and thus it is essential to establish the purity of the B^{12} activity associated with muons. Indeed, analysis showed that a short time-constant component (possibly produced by neutrons) affected the decay curve and would have given a larger apparent reaction rate if undetected.

In order to detect this contaminating activity, it was necessary to study the B^{12} decay curves for a time (~100 msec) long compared to the B^{12} half-life. We chose an "analysis time" t for each curve such that only particles observed after t were deemed acceptable and included in the data analysis. There were two criteria for choosing the value for t for a particular run. First, the value of $T_{1/2}$ computed from the acceptable data had to agree with the known value. This condition was somewhat ineffective in rejecting impure data, because of the well-known deceptiveness of complex decay curves when examined over limited time intervals. Second, the B^{12} amplitude projected back to $t=0$ had to be independent of t. Application of these criteria insured that accepted data included only B^{12} plus random background.

Our result is that the ratio $\lambda_B{}^{\text{total}}/\lambda_{\text{decay}} = (1.55\pm0.06)\times10^{-2}$, where $\lambda_B{}^{\text{total}}$ is the partial absorption rate to bound B^{12}. The statistical and systematic uncertainties are 1.5% and 3.5%, respectively. For $1/\lambda_{\text{decay}}$ we use the μ^+ mean life, 2.211 μsec,[7] to obtain $\lambda_B{}^{\text{total}} = (7.01\pm0.27)\times10^3$ sec^{-1}. Justification for using the μ^+ mean life for $1/\lambda_{\text{decay}}$ is provided by the theory of Überall,[8] which indicates that in the μ-mesonic atom of carbon the muon decays at the same rate as a free muon to within 0.2%.

Our measurement of $\lambda_B{}^{\text{total}}$ includes contributions from (a) direct capture from C^{12} to the ground state of B^{12}, (b) capture to excited bound states of B^{12} which decay via γ-ray emission to the ground state, and (c) production of bound B^{12} from $\mu^- + C^{13} \rightarrow B^{12} + n + \nu$. Only process (a) has been computed, so in order to compare our results to theory we use a result obtained by the Los Alamos group[2] and reduce $\lambda_B{}^{\text{total}}$ by 10% to account for process (b). Process (c), which may contribute as much as 5% to the rate, is ignored. The adjusted partial capture rate to the ground state only is $\lambda_B{}^{\text{gnd}} = (6.31\pm0.24)\times10^3$ sec^{-1}, where no uncertainty for the 10% correction has been included.

Earlier experiments[1-5] yielded values for $\lambda_B{}^{\text{gnd}}\times10^{-3}$ sec^{-1} of (5.9 ± 1.5), (9.05 ± 0.95), (9.18 ± 0.5), (6.8 ± 1.1), and (5.8 ± 1.3), respectively. Our result is consistent with those of references 1, 4, and 5, but there is disagreement beyond quoted errors with the Los Alamos[2] and CERN[3] values. We note first that the Los Alamos result includes an error involving only statistical contributions, and that systematic effects might still be present. There is insufficient information available to permit detailed commentary on the CERN experiment. However, we feel that the precautions taken in our work to exclude spurious time components produced by particles other than muons (pions and neutrons) were necessary.

The various theoretical analyses[9] of this process indicate that $\lambda_B{}^{\text{gnd}}$ may be expected to be $(5.5 \text{ or } 7.5)\times10^3$ sec^{-1} if (a) the vector (g_V) and axial vector (g_A) coupling constants for muon capture are 0.97 and 1.00 times the corresponding constants for β decay (the universal Fermi interaction hypothesis, corrected for momentum transfer effects), and (b) the induced pseudoscalar coupling constant $g_p = +8g_A$. The limiting values indicated for the theoretical $\lambda_B{}^{\text{gnd}}$ will obtain depending on whether the conserved vector current contribution is absent or present. The measurement of $\lambda_B{}^{\text{gnd}}$ provides a single number, which cannot be used to verify all these hypotheses. If (a) and (b) are accepted, one can attempt to draw conclusions about the weak magnetism. Whereas this point of view applied to the CERN results would provide rather strong evidence of the existence of a weak magnetism contribution, our result does not confirm this. Our number is in fact consistent with either the inclusion or omission of the weak magnetism in the interaction. This is partly a result of theoretical uncertainties of 10-15% involving the B^{12} and C^{12} nuclear wave functions, and partly because of the uncertain excited-state contributions to $\lambda_B{}^{\text{total}}$.

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PION-PION INTERACTION FROM THRESHOLD ANOMALIES IN K^+ DECAY

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The importance of the (π,π) interaction has been recently emphasized by many authors and various methods have been conceived for obtaining information on the (π,π) interaction cross sections.[1] It is our purpose to discuss in this note, for this kind of problem, a general method which allows in principle the determination of certain cross sections which are out of reach of direct experimental measurement.

The method is based on the observation of threshold anomalies of the kind of a cusp or rounded step in cross sections for reactions with three particles in the final state.[2,3] The necessary requirements are that we must have (at least) two different reactions with three particles in the final state, that one of the particles is common to both the considered reactions, and that the phenomenon is analyzed by keeping the total energy fixed. In processes of this kind it is very important to have a good energy resolution for the incoming beam; a spread of the initial energy can in fact destroy the threshold effect which usually involves a small portion of the final spectrum. In this respect an ideal situation presents itself in the three-particle decays of an unstable particle whose mass is well determined. As an example we will discuss the decay of the K^+ meson into three pions, for which all the requirements for the appearance of the effect are satisfied[4]:

$$K^+ \to \pi^+ + \pi^- + \pi^+, \qquad \text{(a)}$$

$$K^+ \to \pi^+ + \pi^0 + \pi^0. \qquad \text{(b)}$$

If we fix our attention on the π^+ meson coming from the decay (b) and we vary its kinetic energy E_1 starting from its maximum value, we see that at first there can be no corresponding π^+ from (a) with the same energy, the decay (a) being there energetically forbidden due to the heavier mass of the charged with respect to the neutral pions. By decreasing E_1 and therefore by increasing the relative energy $E_2{}^b$ of the (π^0,π^0) pair, we get to the point $\overline{E}_2{}^b = 2m^+ - 2m^0 \cong 9.2$ Mev where a (π^-,π^+) pair can be created at rest (in its center-of-mass system) in a (π^0,π^0) collision. By increasing $E_2{}^b$ over that value it is by now clear that the decay (a) will also be energetically possible so that $\overline{E}_2{}^b$, or the corresponding $\overline{E}_1$, acts essentially as the threshold energy for the reaction $\pi^0+\pi^0 \to \pi^- + \pi^+$. At $\overline{E}_1$ a threshold effect of the kind of a cusp or rounded step will be observed in the spectrum of the π^+ coming from decay (b) and the size of such an effect will depend critically on the low-energy value of the pion-pion charge exchange cross section. Since at threshold the (π^-,π^+) pair is in a relative S wave, from conservation of parity (here applicable) and total angular momentum, it follows that the threshold effect will appear in the relative S wave of the (π^0,π^0) pair. Further, due to the small relative energy of the (π^0,π^0) pair at the $\pi^-\pi^+$ threshold, the relative waves with $l > 0$ will be negligible, so that the π^+ common to both decays will also be in the S wave since the K^+-meson spin is zero. The details of the phenomenon are described by Eq. (18) of reference 2. When integrated over

angles, that equation reads as follows:

$$\frac{\partial}{\partial|k_{2a}|}\left[k_{2f}^{-1}\frac{dw_f(k_1)}{k_1^2 dk_1}\right] = -\pi^{-1/2}\left(\frac{\mathrm{Im}}{\mathrm{Re}}\right)k_{2f}^{-1}T_f^*(k_1)T_a(k_1)\Theta_{fa}(k_{2a}), \quad f=a,b \tag{1}$$

where only the S waves for the final products of decays (a) and (b) have been taken into account. We have introduced in (1) a k_{2f}^{-1} so that the equation holds also for $f=a$, the decay (a) of the K^+ meson. Both sides of Eq. (1) are evaluated at the threshold energy $\bar{E}_1$. $dw_f(k_1)/k_1^2 dk_1 = |T_f(k_1)|^2$ is the probability per unit time of finding for the decay (f) the particle 1, i.e., the π^+ common to both decays, in the S wave with momentum between $\hbar k_1$ and $\hbar k_1 + d\hbar k_1$, and particles 2 and 3 in the relative S wave with relative momentum $\hbar k_{2f}$ (which is fixed once $\hbar k_1$ is given). The symbol $\left(\frac{\mathrm{Im}}{\mathrm{Re}}\right)$ means that we have to take the imaginary part of the expression on the right-hand side of (1) when decay (a) is possible, the real part when (a) is forbidden. $\Theta_{fa}(k_{2a})$ is the transition amplitude whose squared magnitude is the pion-pion S-wave partial cross section for charge exchange if $f=b$, for scattering of π^- on π^+ if $f=a$.[5]

For $f=b$ we easily obtain from (1):

$$\left(\frac{\partial}{\partial|k_{2a}|}\frac{dw(K^+\to\pi^+\pi^0\pi^0)}{dE_1}\bigg|_a\right)^2+\left(\frac{\partial}{\partial|k_{2a}|}\frac{dw(K^+\to\pi^+\pi^0\pi^0)}{dE_1}\bigg|_b\right)^2 = \pi^{-1}\left(\frac{dw(K^+\to\pi^+\pi^0\pi^0)}{dE_1}\right)\left(k_{2a}^{-1}\frac{dw(K^+\to\pi^+\pi^-\pi^+)}{dE_1}\right)\left[k_{2a}\sigma(\pi^-\pi^+\to\pi^0\pi^0)\right], \tag{2}$$

where $\sigma(\pi^-\pi^+\to\pi^0\pi^0)$ is the pion-pion charge exchange cross section integrated over angles. Both sides of Eq. (2) are evaluated at the $\pi^-\pi^+$ threshold; the first parenthesis on the left-hand side is evaluated as the $\pi^-\pi^+$ threshold is approached from above, the second as the threshold is reached from below. Since $\sigma(\pi^-\pi^+\to\pi^0\pi^0)$ goes to infinity linearly in the limit $k_{2a}\to 0$, a k_{2a} has been placed to counterbalance this effect. Similarly $dw(K^+\to\pi^+\pi^-\pi^+)/dE_1$ goes to zero linearly in the same limit, so k_{2a}^{-1} makes it nonzero there.

For $f=a$, using the optical theorem,

$$\mathrm{Im}\Theta_{aa}(k_{2a}) = [k_{2a}/(4\pi)^{1/2}]\sigma_{\pi^-\pi^+}^{\mathrm{tot}},$$

we get[6]

$$\frac{\partial}{\partial k_{2a}}\left(k_{2a}^{-1}\frac{dw(K^+\to\pi^+\pi^-\pi^+)}{dE_1}\right) = -\frac{1}{2\pi}\left(k_{2a}^{-1}\frac{dw(K^+\to\pi^+\pi^-\pi^+)}{dE_1}\right)\times\left(k_{2a}\sigma_{\pi^-\pi^+}^{\mathrm{tot}}\right), \tag{3}$$

where again both members are evaluated at the $\pi^-\pi^+$ threshold. $\sigma_{\pi^-\pi^+}^{\mathrm{tot}}$, the zero-energy total cross section for the channel $\pi^-\pi^+$, is the pion-pion charge-exchange cross section plus the cross section for the scattering of π^- on π^+. The latter goes to a constant in the limit $k_{2a}\to 0$, while the former goes to infinity. Therefore, we can replace on the right-hand side of (3) $\sigma_{\pi^-\pi^+}^{\mathrm{tot}}$ with the charge-exchange cross section $\sigma(\pi^-\pi^+\to\pi^0\pi^0)$.

We see, therefore, that if a detailed measurement of the π^+ distribution is performed around the $\pi^-\pi^+$ threshold, one can determine in principle, with the use of either Eq. (2) or Eq. (3), the pion-pion charge-exchange cross section at zero energy. The applicability of the method is naturally bound to the statistics available and to the energy resolution attainable in decays (a) and (b). Good results will be obtained if refined measurements will be done in the energy region considered and if the second derivative of $k_{2f}^{-1}dw_f(k_1)/dE_1$ will turn out to be negligible over a relatively large range of energies, so that the slope of this function at the $\pi^-\pi^+$ threshold can be determined with good accuracy. Since the statistics

available at present around the $\pi^-\pi^+$ threshold are poor and consequently the discussed threshold effect is not seen in the experimental π^+ distribution, we cannot give here any quantitative estimate of $\sigma(\pi^-\pi^+\to\pi^0\pi^0)$. We note, however, that the threshold effect can in general be quite visible for this kind of interaction. A qualitative estimate of its size can be given by using Eq. (2). Assuming that

$$(k_{2b}/k_{2a})dw(K^+\to\pi^+\pi^-\pi^+)/dE_1 \cong 3dw(K^+\to\pi^+\pi^0\pi^0)/dE_1,$$

as the experimental results on the integrated spectra suggest, taking for $(k_{2a}/k_{2b})\sigma(\pi^-\pi^+\to\pi^0\pi^0)$ a value around 10 mb as expected from other work,[1] and supposing that the above-mentioned second derivative is appreciably small over a range of a few Mev around the $\pi^-\pi^+$ threshold, we get a threshold effect of 5-10% in the π^+ distribution for decay (b) of the K^+ meson. Obviously, this argument can be reversed to give a qualitative estimate of $\sigma(\pi^-\pi^+\to\pi^0\pi^0)$ once the threshold effect is found experimentally.

[1]See, for example, J. A. Anderson, P. G. Burke, D. D. Carmony, and N. Schmitz, Proceedings of the 1960 Annual Conference on High-Energy Physics at Rochester (Interscience Publishers, New York, 1960), p. 58; Yu. A. Batusov, S. A. Bunyatov, V. M. Sidorov, and V. A. Yarba, ibid., p. 79; and the copious references contained in these two reports.

[2]L. Fonda and R. G. Newton, Phys. Rev. 119, 1394 (1960).

[3]L. I. Lapidus and Chou Kuang-chao, J. Exptl. Theoret. Phys. (U.S.S.R.) 39, 364 (1960) [translation: Soviet Phys.—JETP 39(12), 258 (1961)].

[4]The same procedure will apply also to the photoproduction of two pions on hydrogen and in other cases.

[5]Coulomb effects cover a very small energy region around $\bar{E}_1$ and have been neglected.

[6]Equations similar to (2) and (3) have been obtained for two-particle→two-particle reactions by R. G. Newton, Phys. Rev. 114, 1611 (1959); and by L. Fonda and R. G. Newton, Nuovo cimento 14, 1027 (1959).

π-Λ RESONANCE AND THE SIGMA HYPERON

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A resonance in the π-Λ system has been reported at a center-of-mass total energy of 1385 ± 15 Mev.[1,2] In this note we would like to consider a possible connection between this resonance and the sigma hyperon, in the case in which the latter particle is hypothesized to be a bound state of a pion and a Λ with binding energy of ~ -65 Mev.[3]

Dalitz and Tuan[4] have shown that the coupling of the π-Λ system to the $\bar{K}$-nucleon system at threshold can lead to a resonance in π-Λ scattering at some energy below the $\bar{K}$-nucleon threshold if the so-called $(a-)$ solution[4] turns out to describe the $\bar{K}$-nucleon system at moderate energies above threshold. In an approximation in which we neglect π-Σ production in the isotopic spin one state in K^--p reactions at threshold,[5] we may write a formula for the π-Λ S-wave phase shift, δ, according to the zero-range formulation of Dalitz and Tuan[4]:

$$(q/E)\cot\delta = (q_t/E_t)\cot\delta_t \times\{[1+\kappa(a+b\tan\delta_t)]/[1+\kappa(a-b\cot\delta_t)]\}. \quad (1)$$

In this formula q and E are the center-of-mass momentum and total energy, respectively, in the π-Λ system; the subscript t denotes an evaluation of the various quantities at the $\bar{K}$-nucleon threshold; κ is defined in terms of q by the following equation:

$$(q^2+m_\pi{}^2)^{1/2}+(q^2+m_\Lambda{}^2)^{1/2} = (-\kappa^2+m_K{}^2)^{1/2}+(-\kappa^2+m_p{}^2)^{1/2}, \quad (2)$$

where the various masses are denoted by an appropriate subscript. The a and b are taken to be the parameters of the $(a-)$ solution of Dalitz,[6] $a=-1.09$ fermis, $b=0.20$ fermi. We note that in the limit of $b\to0$ (i.e., the coupling of the π-Λ system to the $\bar{K}$-nucleon system at threshold going to zero), we have the zero-range formula:

$$(q/E)\cot\delta = (q_t/E_t)\cot\delta_t = \text{constant}. \quad (3)$$

The phase δ_t depends on the various dynamic properties of the π-Λ system (the various mech-

anisms giving rise to the forces between a pion and a Λ), of which the coupling to the $\bar{K}$-nucleon system may be but one. Implicit in the following discussion is the assumption that there is more to the dynamics of the π-Λ system than is contained in its coupling to the $\bar{K}$-nucleon system at threshold. In particular, the exchange of two pions through the vertices $\Lambda \to \Lambda + 2\pi$ and $\pi + 2\pi \to \pi$ will give rise to an attraction if the two vertices have positive couplings. The zero-range formula for $q\cot\delta$ may reasonably be expected to be a fair approximation since presumably the longest range force between a pion and a Λ is mediated by the exchange of two pions (the effective range is likely to be less than $1/2m_\pi$), while q varies from 0 to $q \sim 1.5\, m_\pi$ at resonance. We would like to determine δ_t by requiring that the scattering resonance be at its observed position, ~1385 Mev. We then ask what is $\lim_{q\to 0} q\cot\delta = -1/A$, as determined by Eq. (1). Here A is the scattering length in the π-Λ system. We find $\delta_t \sim 154.5°$ or $-25.5°$ and $-1/A \sim +82m_\pi$. It is of interest to note that in the limit as $b \to 0$ (this corresponds to a very narrow π-Λ scattering resonance[4]) under the assumption that δ_t remains near the value determined above, $-1/A \to -3.47m_\pi$. The negative sign in the latter result is of particular interest, for it allows for the following possible interpretation of the scattering length behavior: In the limit of $b \to 0$, the zero-energy π-Λ S-wave phase shift starts at π and the system exhibits a scattering length characteristic of an attractive potential more than deep enough to have one bound state, but perhaps not deep enough to have a second bound state; for $b = 0.20$, the coupling of the π-Λ system to the $\bar{K}$-nucleon system at threshold results in an even stronger attraction, as indicated by a change in the scattering length from a positive value, $A \sim 1/3.47m_\pi$, to a very small negative value. The phase shift starts at π, goes through a resonance at $3\pi/2$, and approaches $\delta_t \sim -25.5°$ at the $\bar{K}$-nucleon threshold. The suggested bound state would be interpreted as the sigma hyperon; the quantum numbers of the π-Λ scattering resonance should be identical with those of the sigma hyperon, in other words the Y^* (the π-Λ resonance) and Σ should have the same spin, relative parity, strangeness, and isotopic spin. It is perhaps interesting to note that if the mass of the Y^* were at ~1395 Mev, we would obtain $\delta_t \sim 127.5°$ or $-52.5°$ and A would go from $\sim 1/3.43m_\pi$ to $\sim 1/1.27m_\pi$ as b goes from 0.20 to zero. The value, $A \sim 1/1.27m_\pi$, is not too far from the value one would obtain by assuming, in this case, that the zero-energy wave function was not too different from the bound-state wave function, $A \sim R$, where $R \sim 1/1.07m_\pi$, as determined by a binding energy of -65 Mev for the sigma hyperon. This somewhat academic remark is made simply because of the apparent tendency for the experimental resonance position to rise slightly and we would like to see the modification of the argument in this case. For a resonance at ~1385 Mev, the value of $\delta_t \sim -25.5°$ causes a relatively slight distortion of the symmetry of the resonance shape; however, for a resonance at ~1395 Mev, the value of $\delta_t \sim 127.5°$ or $-52.5°$ would imply a considerably slower falloff on the high-energy side of the resonance than on the low-energy side.

It is clear that an alternative interpretation of the zero-energy behavior of $q\cot\delta$ is possible, namely, that the extremely small negative scattering length, $A \sim -1/82m_\pi$, implies a very weak attraction (the zero-energy S-wave π-Λ phase shift starts at zero), which in the limit of $b \to 0$ goes to a repulsion characterized by $A \sim 1/3.47m_\pi$. However, the latter limit includes the assumption that δ_t remains close to its value as determined by the position of the π-Λ resonance with $b = 0.20$. Since in this case there would seem to be very little dynamics to the π-Λ system other than its coupling to the $\bar{K}$-nucleon system at threshold, δ_t probably cannot be considered to remain reasonably constant as $b \to 0$, but rather δ_t may $\to 0$ ($A \to 0$) as $b \to 0$.

The position of the resonance is not very sensitive to the introduction of dynamics in the π-Λ system through a nonzero δ_t; for $\delta_t = 0$ or π, the resonance is at ~1378 Mev. However, the zero-energy scattering length is quite sensitive to δ_t (for $\delta_t = 0$ or π, $A \sim -1/3.27m_\pi$). In this sense, the precise position of the resonance, which is predicted in general for $b \neq 0$,[4] may well reflect additional dynamics in the π-Λ system. It is interesting that an extrapolation of the ($a+$) solution[4] of Dalitz to zero energy in the π-Λ system (with $\delta_t = 0$) would imply a weak repulsion. If we assume in the bound-state model that the coupling of the π-Λ system to the $\bar{K}$-nucleon system should contribute an attraction, then the existence of the bound state would imply the ($a-$) solution.

In conclusion we state that the value of the above rather speculative remarks could be most decisively tested by a determination and comparison of the spins and relative parities of the π-Λ resonance and the sigma hyperon. Since

the spin of sigma hyperon is $\frac{1}{2}$, the spin of the π-Λ resonance must be $\frac{1}{2}$. It has been suggested[3] that the bound-state hypothesis is most attractive for an S-state π-Λ system. This implies that the Σ-Λ relative parity is odd[7] and that the π-Λ resonance is in the S wave. Given certain assumptions, the bound-state model makes a prediction[7] with regard to the relative sign of the asymmetry parameter, α_Λ, in the decay $\Lambda \to \pi^- + p$, and the asymmetry parameter, α_0, in the decay $\Sigma^+ \to \pi^0 + p$, namely $\alpha_\Lambda \alpha_0 \sim +1$ for an $S_{1/2}$ bound state (a $P_{1/2}$ bound state would give a negative relative sign). The positive relative sign is to be contrasted with the negative relative sign predicted by certain theories[8,9] of strong and weak interactions which assume even Σ-Λ relative parity. These latter theories also predict $\alpha_\Lambda \alpha_\Xi \sim +1$, where α_Ξ is the asymmetry parameter in the decay $\Xi \to \pi^- + \Lambda$. This result is in contradiction to recent experiment.[10]

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[1]M. H. Alston et al., Phys. Rev. Letters 5, 520 (1960).

[2]Private communication from A. Rosenfeld.

[3]S. Barshay and M. Schwartz, Phys. Rev. Letters 4, 618 (1960).

[4]R. H. Dalitz and S. F. Tuan, Ann. Phys. 10, 307 (1960).

[5]The (Σ/Λ) ratio in the isotopic one state is apparently at least 0.2 at threshold, but falls off sharply below the $\bar{K}$-nucleon threshold, the region in which we will be applying formula (1).

[6]R. H. Dalitz, Phys. Rev. Letters 6, 239 (1961).

[7]S. Barshay, Phys. Rev. Letters 1, 97 (1958).

[8]S. Treiman, Nuovo cimento 15, 916 (1960).

[9]A. Pais, Nuovo cimento 18, 1003 (1960).

[10]W. B. Fowler et al., Phys. Rev. Letters 6, 134 (1961).

ELECTRODYNAMIC PROPERTIES OF BARYONS IN THE UNITARY SYMMETRY SCHEME

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Gell-Mann[1] recently introduced a theory of strong interactions involving a new symmetry, called "unitary symmetry." The principal purpose of this note is to use this symmetry to express the magnetic moments of all the baryons in terms of those of the neutron and proton. We also derive a relation among baryon electromagnetic mass splittings.

The unitary symmetry scheme proposes that the elementary particles may be represented as tensors in a three-dimensional (generalized isospin) space, and that the strong interactions are invariant under unitary transformations in this space. In particular, the eight baryons form the components of a traceless matrix ψ,

$$\psi = \begin{pmatrix} -(\frac{2}{3})^{1/2}\Lambda & p & n \\ \Xi^- & (\frac{1}{6})^{1/2}\Lambda + (\frac{1}{2})^{1/2}\Sigma^0 & \Sigma^- \\ \Xi^0 & \Sigma^+ & (\frac{1}{6})^{1/2}\Lambda - (\frac{1}{2})^{1/2}\Sigma^0 \end{pmatrix}, \tag{1}$$

while the seven known pseudoscalar mesons (plus a predicted new pseudoscalar meson, χ^0) form the components of a traceless Hermitian matrix ϕ,

$$\phi = \begin{pmatrix} -(\frac{2}{3})^{1/2}\chi^0 & K^+ & K^0 \\ K^- & (\frac{1}{6})^{1/2}\chi^0 + (\frac{1}{2})^{1/2}\pi^0 & \pi^- \\ \bar{K}^0 & \pi^+ & (\frac{1}{6})^{1/2}\chi^0 - (\frac{1}{2})^{1/2}\pi^0 \end{pmatrix}. \tag{2}$$

The scheme also proposes the existence of eight vector mesons which transform in the same way as ϕ. Although of great importance in Gell-Mann's theory, they will not be described here, for their symmetric interactions do not affect our conclusions.

If the strong interactions are to be invariant under unitary symmetry, the possible forms of the Lagrangian density are (assuming *ps-ps* meson-nucleon coupling)

$$L = \mathrm{tr}\bar{\psi}(i\partial_\mu \gamma^\mu - m_0)\psi + \mathrm{tr}(\tfrac{1}{2}\partial_\mu \phi \partial^\mu \phi - \tfrac{1}{2}M_0^2\phi^2) + g\,\mathrm{tr}(\bar{\psi}\gamma_5\psi\phi) + g'\,\mathrm{tr}(\bar{\psi}\gamma_5\phi\psi) + L', \tag{3}$$

where $\bar{\psi}$ is $\psi^\dagger\gamma_0$, tr is the ordinary matrix trace over the generalized isospin space, m_0 is a common baryon bare mass, and M_0 is a common meson bare mass.

There are two terms in L':

(1) A mysterious unknown interaction, weaker than the strong interactions, that breaks the unitary symmetry and causes us to observe the baryons as a singlet, two doublets, and a triplet, rather than as a completely degenerate octet. We know nothing about this interaction—it may be something as simple as a difference in baryon bare masses—and can predict nothing about its effects. Our conclusions are approximations which are exact only in the absence of this mass-splitting interaction. We hope that the mass-splitting interaction is sufficiently weak so that our approximations are good ones; this is the same hope that is fundamental to any quantitative prediction made on the basis of any "higher symmetry" scheme, like global symmetry.

(2) Electromagnetism. In contrast to the mass-splitting interaction, we know the form of the electromagnetic interaction exactly: In our notation it is

$$L_{\text{em}}' = eA^\mu \operatorname{tr}(\bar{\psi}\gamma_\mu[\psi, Q] + i\partial_\mu\phi[\phi, Q]) - \tfrac{1}{2}e^2A^\mu A_\mu \operatorname{tr}([\phi, Q][\phi, Q]), \quad (4)$$

where Q is the real traceless diagonal matrix,

$$Q = \tfrac{1}{3}\begin{pmatrix} -1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & -1 \end{pmatrix}. \quad (5)$$

Without the mass-splitting interactions, the only departure from unitary symmetry is through the appearance of Q in the electromagnetic interaction. To first order in electromagnetism, but to all orders in the symmetric interactions, the electromagnetic vertex function must be a linear homogeneous invariant function of Q; thus,

$$\begin{aligned}\langle\bar{\psi}|A_\lambda|\psi\rangle = {} & \mu_1(q^2)q_\nu \operatorname{tr}(\bar{u}\sigma_{\lambda\nu}vQ) \\ & + \mu_2(q^2)q_\nu \operatorname{tr}(\bar{u}\sigma_{\lambda\nu}Qv) \\ & + e_1(q^2) \operatorname{tr}(\bar{u}\gamma_\lambda vQ) \\ & + e_2(q^2) \operatorname{tr}(\bar{u}\gamma_\lambda Qv),\end{aligned} \quad (6)$$

where u and v are the incoming and outgoing baryon spinors (they are also traceless 3×3 matrices in generalized isospin space), and q_μ is the 4-momentum transfer. Terms in tr$Q$ are absent from (6) because we have chosen Q to be traceless.

The values of e_1 and e_2 at $q^2=0$ are determined by the baryon charges; their higher derivatives yield hyperon form factors which are not likely to be measured soon. Therefore, we fix our attention on the μ_i at $q^2=0$. These give, in principle, nine quantities, the eight baryon magnetic moments and the matrix element for $\Sigma^0 \to \Lambda + \gamma$.

Expanding (6) in terms of (1), we find for the magnetic moments,

$$\mu(\Sigma^+) = \mu(p), \quad (7)$$

$$\mu(\Lambda) = \tfrac{1}{2}\mu(n), \quad (8)$$

$$\mu(\Xi^0) = \mu(n), \quad (9)$$

$$\mu(\Xi^-) = \mu(\Sigma^-) = -[\mu(p) + \mu(n)], \quad (10)$$

$$\mu(\Sigma^0) = -\tfrac{1}{2}\mu(n), \quad (11)$$

and for the mixed moment (the multiple of $\bar{\Sigma}^0\sigma_{\mu\nu}\Lambda F_{\mu\nu}$ + H. c.) responsible for the decay $\Sigma^0 \to \Lambda + \gamma$,

$$\mu_m = \tfrac{1}{2}\sqrt{3}\mu(n). \quad (12)$$

These equations contain one relation that is independent of the unitary symmetry scheme,

$$\mu(\Sigma^0) = \tfrac{1}{2}[\mu(\Sigma^+) + \mu(\Sigma^-)].$$

This is known to be a consequence of isospin invariance alone.[2]

In the same manner we may consider the electromagnetic corrections to the baryon masses. In the absence of the principal mass-splitting interactions, but in the presence of all symmetric strong interactions, the induced electromagnetic mass difference must be an invariant function of Q. The most general such expression is

$$\begin{aligned}\langle\bar{\psi}|\delta m|\psi\rangle = {} & \delta_1 \operatorname{tr}(\bar{u}Qu) + \delta_2 \operatorname{tr}(\bar{u}uQ) \\ & + \delta_3 \operatorname{tr}(\bar{u}QuQ) + \delta_4 \operatorname{tr}(\bar{u}u).\end{aligned} \quad (13)$$

Terms involving Q^2 do not explicitly appear because they are eliminated by the relation $9Q^2 = 2 + 3Q$. The last term in (13) is an addition to the common baryon mass and may be ignored. So, although there are four independent electromagnetic baryon mass splittings, there are only three significant independent parameters in (13). Thus we may obtain a condition on the baryon masses,[3]

$$m(\Xi^-) - m(\Xi^0) = m(\Sigma^-) - m(\Sigma^+) + m(p) - m(n). \quad (14)$$

Gell-Mann[1] has suggested that, in addition to invariance under unitary symmetry, the strong interactions may be invariant under a discrete symmetry, called R. In our notation, R corresponds to replacing every matrix by its transpose. This does not affect the free Lagrangian, but interchanges the two kinds of ps-ps couplings. Thus R invariance demands that $g=g'$.

The electric current is odd under R. Thus we can make the electromagnetic interactions R invariant by extending the definition of R so that A_μ is also odd under R. Should the strong interactions be R invariant as well, it follows in our approximation that $\mu_1=-\mu_2$ and $\delta_1=\delta_2$. This yields the manifestly unacceptable results that both the neutron magnetic moment and the charged Σ mass difference vanish.[4] We consider this sufficient grounds for abandoning the R invariance.

*National Science Foundation Predoctoral Fellow.

[1]M. Gell-Mann, Phys. Rev. (to be published). Although our notation differs from that of Gell-Mann, we study the same symmetries as he does. Gell-Mann emphasizes that his symmetry scheme may be seriously broken; experiment indicates that $K\bar{N}\Lambda$ and $K\bar{N}\Sigma$ interactions are much weaker than $\pi\bar{N}N$ interactions, in conflict with the requirements of unitary symmetry.

[2]R. Marshak, S. Okubo, and G. Sudarshan, Phys. Rev. 106, 599 (1957).

[3]Equation (14) gives a predicted value for $m(\Xi^-)-m(\Xi^0)$ of 5.3 ± 0.2 Mev. The observed value is 7.4 ± 8.0 Mev. We use the data tabulated by W. H. Barkas and A. H. Rosenfeld, Proceedings of the Tenth Annual International Rochester Conference on High-Energy Physics, 1960 (Interscience Publishers, Inc., New York, 1960), p. 877.

[4]It is easy to see this is not true if we abandon R invariance. In this case, we can, for example, set $g'=0$. Then the only interaction of the neutron with charged mesons is through $g(\bar{n}\gamma_5 p\pi^- + \bar{p}\gamma_5 n\pi^+)$. This gives a nonvanishing moment in order eg^2.

POSSIBLE EFFECT OF COLLECTIVE CORRELATION BETWEEN VACUUM NUCLEONS IN PION PHYSICS

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Recently, investigations into interacting systems of many particles have made great advances, and for Fermi systems it has been found that, if the interaction is predominantly attractive, a strong collective correlation arises and the system exhibits properties very different from the free one.[1,2] It seems possible that this effect can play an important role in pion physics also, since the internucleon force between vacuum nucleons due to pions is predominantly attractive.[3] Thus, it is expected that the structure of the vacuum would be very much altered by taking this effect into account, and that the processes in which the properties of the vacuum appear explicitly, such as the formation or the annihilation of nucleon pairs, would be greatly modified.[4]

The vacuum is usually defined as the eigenstate of the total Hamiltonian with the lowest energy eigenvalue. For the pion-nucleon system, if there is no interaction, this is realized by filling all negative-energy levels of the nucleons with all positive levels unoccupied, as given by Dirac. If an attractive interaction exists, however, this is no longer the lowest energy state. Instead, the lowest energy state is realized by moving some of the nucleons in negative-energy states to positive-energy states. This is because although the energy of the single nucleon is increased by 2κ when the nucleon is moved from a negative-energy state to a positive-energy state, where κ is the nucleon mass,[5] the potential energy of the total system can be decreased by doing so because attractive potentials, which had no effect when all negative levels were filled due to the Pauli principle, become effective. Thus, the total energy can be decreased by moving some of the nucleons in negative-energy states to positive-energy states, and can take a minimum value at a distribution different from the free one. This is exactly the same as the situation in which a superconducting state is realized by moving some of the electrons below the Fermi energy above it. So, if we want to define the vacuum as the eigenstate of the total system with the lowest energy eigenvalue, we must take this state as the true vacuum for the pion-nucleon system.

Of course, in order that such a situation be realized, the attractive potential must be sufficiently strong that the decrease of the potential

energy can exceed 2κ. If we assume most simply that the attractive potential is given by the Yukawa potential $-g^2e^{-\mu r}/r$, where μ is the pion mass, it can be shown that this condition is given by[6,7]

$$g^2 \geq \kappa/K, \tag{1}$$

where K is the cutoff momentum. At first sight, it seems impossible to satisfy (1) if we take $K \simeq \kappa$, since g^2 is expected to be of order 0.1.

It should be noted, however, that it is not necessary to identify κ with its observed value. As emphasized by Nambu and Jona-Lasinio,[8] there exists a special mechanism for producing a large nucleon mass if the internucleon force is attractive. This is based on the well-known property of the Fermi system with attractive interaction that it has a gap in its excitation spectrum, which is a very general feature of such a system. So, we can assume that the bare mass is small, at most of order μ, and that the actual mass is largely of interaction origin. This assumption is very tempting, because in this scheme the large mass of the nucleon can be understood in connection with its strong interaction with pions.

If we assume that the bare nucleon mass is small, then the energy of the nucleon is small for small $\vec{k}$, where $\vec{k}$ is the momentum of the nucleon, and for these nucleons the decrease of the potential energy can exceed the increase of the single-nucleon energy. So, we can find a critical value of $|\vec{k}|$, k_C, such that for $|\vec{k}| < k_C$, the energy of the total system is decreased by moving nucleons with such momentum from negative-energy states to positive-energy states. Thus, in this region, the lowest energy state is actually realized by moving some of the nucleons from negative-energy states to positive-energy states. In the region with $|\vec{k}| > k_C$, the situation is the same as in the free case, since there the energy of the nucleon is large and it is impossible that the decrease of the potential energy exceeds the increase of the single-nucleon energy. Thus, as the first approximation, we may picture the vacuum as the state in which nucleons in negative-energy states with momentum $|\vec{k}| < k_C$ go up partly to positive-energy states. The number of these nucleons is, of course, zero in the weak-coupling limit, and it can be shown that, if we adopt a simplified model in which the Fourier components of the attractive potential $v(\vec{k}_1, \vec{k}_2)$ are constant for $|\vec{k}_1|$ and $|\vec{k}_2| < k_C$ and zero otherwise, just one half of the nucleons in negative-energy states go up to positive-energy states in the strong-coupling limit.[7] It can also be shown that k_C is of the order of μ in this approximation.[7]

We may expect that essentially the same situation should exist when some nucleons or antinucleons are present. Thus, it is expected that physical nucleons (antinucleons) are always accompanied by nucleon pairs which are formed by some of the negative-energy nucleons going to positive levels for exactly the same reason as in the vacuum case. This expectation is justified in the same approximation as before, at least for one-particle states.[7,9] The observed mass of the nucleon (antinucleon) is defined as the energy of these states relative to that of the vacuum.

It is obvious that such alteration of the vacuum and the physical nucleon (antinucleon) states modifies pion physics in various ways. Consider, for example, the vertex corresponding to the production of a nucleon-antinucleon pair by a pion, which is the transition of a nucleon from a negative-energy state to a positive-energy state. The matrix element for this process is decreased for two reasons. The first is that the density of the negative-energy nucleons is reduced. The second is that positive-energy levels are partly filled. This reduces the matrix element by the Pauli principle. It can be shown that the reduction factor $f(\vec{k}_1, \vec{k}_2)$ is given by

$$f(\vec{k}_1, \vec{k}_2) = [(M+r)/2M]^{\epsilon(\vec{k}_1)+\epsilon(\vec{k}_2)}, \tag{2}$$

in the same approximation as before, where $\vec{k}_1$ and $\vec{k}_2$ are the momenta of the nucleon before and after the transition,[7] and $\epsilon(\vec{k})$ is zero if $|\vec{k}| > k_C$ and $\frac{1}{2}$ if $|\vec{k}| < k_C$. M is one half of the total number of levels with $|\vec{k}| < k_C$, and r is one half of the difference of the number of occupied levels in positive- and negative-energy states.[10] It can also be shown that the same reduction factor appears in all vertices in which a nucleon (antinucleon) makes a transition from $\vec{k}_1$ to $\vec{k}_2$, if we neglect terms of order $1/M$ as compared to unity.[7]

As an example of this effect, we shall calculate the scattering of S-wave pions by nucleons in Ps-Ps theory. In Tamm-Dancoff approximation including up to two pions and one nucleon pair, the scattering length in the $T = 3/2$ state, a_3, is given by[7]

$$a_3 = [(M+r)/2M]^3 a_3^{(0)}, \tag{3}$$

where $a_3^{(0)}$ is the value which does not include the effect of collective correlation. $a_3^{(0)}$ calculated thus far has turned out to be too small. ($a_3^{(0)} \simeq -0.4$,[11] as compared to the experimental

value -0.11.[12]) The introduction of the effect of collective correlation improves the situation greatly, and we see that agreement with experiment can be obtained by taking $r/M \simeq 0.3$.[13]

Thus, we can say that if we assume that the bare nucleon mass is small as Nambu did, the effect of collective correlation exists and it plays an important role in low-energy pion physics. Unfortunately, the techniques developed thus far in many-particle physics are not sufficient to treat this problem, and the best we can do today cannot be much more than a plausible discussion. This is because, as a many-particle system, the pion-nucleon system is much more complicated than the electron-phonon system in metals. To get a sensible answer, therefore, it is essential to carry out the calculation in such a way that the symmetry properties or the conservation laws, such as the charge conjugation invariance or the conservation law of nucleon number, are always maintained. None of the existing methods, however, satisfy this requirement. Also, it is not easy to incorporate this mechanism into the formalism of modern field theory. If it turns out that such an effect really exists, however, it means that it would be important to develop a nonperturbative method for the better understanding of pion physics, because the vacuum or physical nucleon (antinucleon) states considered here can never be reached from the free ones by the perturbation, just as the superconducting state can never be reached from the normal state by the perturbation.

The author wishes to express his sincere gratitude to the members of the theoretical division of the School of Physics for valuable discussions and criticism.

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*On leave of absence from the Nihon University, Tokyo, Japan.

[1]J. Bardeen, L. N. Cooper, and J. R. Schriefer, Phys. Rev. 108, 1175 (1957).

[2]N. N. Bogoliubov, J. Exptl. Theoret. Phys. (U.S.S.R.) 34, 58 (1958) [translation: Soviet Phys.-JETP 7, 41 (1958)].

[3]We may expect that the nuclear force between nucleons in positive-energy states is predominantly attractive, since they form nuclei. Then the nuclear force between nucleons in negative-energy states must also be predominantly attractive, because they must be equal by charge conjugation invariance.

[4]A preliminary calculation, using the Bardeen-Cooper-Schrieffer approximation, has been published [O. Hara, Progr. Theoret. Phys. (Kyoto) 24, 495 (1960)].

[5]We neglect the kinetic energy of the nucleon as compared to the rest energy, expecting a cutoff at an ordinary order of magnitude.

[6]Such a condition does not appear in the case of superconductivity. This is because in this case there is no gap corresponding to 2κ in the spectrum of the free system. The superconducting state is realized, however weak the interaction is, so long as the latter is attractive.

[7]O. Hara (to be published).

[8]Y. Nambu and G. Jona-Lasinio, Phys. Rev. 122, 345 (1961).

[9]Thus far, the cases where two nucleons or two antinucleons with opposite momenta exist have not been treated successfully. This defect is, however, not serious in pion-nucleon scattering discussed here.

[10]$r \to 0$ in the strong-coupling limit and $r \to M$ in the weak-coupling limit. Thus, $f(\vec{k}_1, \vec{k}_2)$ is always less than unity and approaches unity in the weak-coupling limit.

[11]F. J. Dyson, M. Ross, E. E. Salpeter, S. S. Schweber, M. K. Sundaresan, W. M. Visscher, and H. A. Bethe, Phys. Rev. 95, 1644 (1954).

[12]J. Orear, Phys. Rev. 96, 176 (1954).

[13]We do not assert that the discrepancy between the existing theories and experiment should be explained by this effect alone. There are several other factors which must be considered. For example, the introduction of pion-pion interaction improves the situation to some extent [K. Kawarabayashi and H. Miyazawa, Progr. Theoret. Phys. (Kyoto) 23, 490 (1960)]. But it seems that the effect of pion-pion interaction is not enough to get complete agreement [M. Sugawara and A. Kanazawa, Phys. Rev. 115, 1310 (1959)], and we feel that the effect of collective correlation should also be considered for the complete understanding of the S-wave pion-nucleon scattering.

ERRATA

K-π RESONANCE AND THE LIFETIME OF THE K' PARTICLE. Mirza A. Baqi Bég and Paul C. DeCelles [Phys. Rev. Letters 6, 145 (1961)].

A numerical error in Eq. (2) affects many of the subsequent equations as well as the tentative indentification of the spin of the π-K resonance. The equations may be corrected throughout the Letter by replacing τ_S (τ_V) by $3\tau_S$ ($3\tau_V$) everywhere it occurs. Accordingly, the paragraph containing (11) should be replaced by "Alternatively, a precise knowledge of σ and the resonance width, Γ, could distinguish between the two alternatives. With the available data,

$$\sigma(K^- + p \to \bar{K}^0 + \pi^- + p)$$
$$= 2\sigma(K^- + p \to K^- + \pi^0 + p) \approx (1 \pm 0.42) \text{ mb},$$

and thus

$$0.298 < \tau_S \times 10^{23} < 0.729,$$
$$3.76 < \tau_V \times 10^{23} < 9.20, \qquad (11)$$

whereas $2.38 < (1/\Gamma_{\text{exp}}) \times 10^{23} < 3.58$. Therefore, present experimental evidence supports the KV alternative."

We wish to thank several colleagues who called our attention to this error.

TWO-PARAMETER APPROXIMATION TO S-WAVE SCATTERING. Daniel M. Greenberger and B. Margolis [Phys. Rev. Letters 6, 310 (1961)].

In formula (3) replace a by $2a$. In the paragraph below formula (3) replace z by ν everywhere.

ABSTRACTS

In this section are printed the abstracts of Articles that have been forwarded to The American Institute of Physics for publication in THE PHYSICAL REVIEW. In quoting information obtained from this section before the appearance of the corresponding Article, reference should be made to "Physical Review (to be published)" rather than to this Journal.

ROTATIONAL MULTIPLETS IN THE SPECTRUM OF THE EARTH. C. L. Pekeris, Z. Alterman, and H. Jarosch, Department of Applied Mathematics, The Weizmann Institute, Rehovot, Israel (Received February 10, 1961).

The doublets in the spectrum of the free oscillations of the earth which have been observed on the gravimetric (UCLA) and strain-meter (Pasadena) records of the Great Chilean Earthquake of May 22, 1960, are interpreted as multiplets arising from the rotation of the earth. The phenomenon is similar to the Zeeman effect, and is indeed a realization of the mechanical analog from which Larmor deduced the "Larmor precession" in his interpretation of the Zeeman effect. A first-order perturbation calculation yields the result that the degenerate frequency $\sigma_0(n)$ in the absence of rotation is resolved by a slow rotation into $(2n+1)$ lines σ_n^m, given by

$$\sigma_n^m = \sigma_0(n) + m\tau(n)\omega, \quad -n \leq m \leq n$$

where ω denotes the angular velocity of rotation of the earth, and m is the azimuthal number of the wave function. $\tau(n)$ is determinable from the zero-order solution in the case of spheroidal oscillations, and is equal to $[n(n+1)]^{-1}$, in the case of purely torsional oscillations. The relative intensities within the quintet $n=2$ and the septet $n=3$ have been determined for an observing station at Los Angeles, on the assumption of an explosive point source at the earthquake focus in Chile. The strongest lines should be the pair $m=\pm 1$ for $n=2$, and the pair $m=\pm 2$ for $n=3$. These agree in separation with the pairs observed on the strain meter and with the gravimetric pair at $n=3$, but less so with the gravimetric pair at $n=2$. There are indications in the strain-meter spectrum for $n=3$ of a weaker line at $m=0$, while the other lines are theoretically of an intensity not exceeding the background noise. The separation in the observed gravimetric doublet for the first overtone of $n=3$ agrees with the interval of the strongest pair $m=\pm 2$. The intensities of the lines in the rotational multiplets of the components of displacement for an observing station at Palisades, New York, have also been determined.

LONG-RANGE CORRELATIONS IN A CLOSED SYSTEM WITH APPLICATIONS TO NONUNIFORM FLUIDS. J. L. Lebowitz, Graduate School of Science, Yeshiva University, New York, New York, and J. K. Percus, Institute of Mathematical Sciences, New York University, New York, New York (Received November 23, 1960).

We investigate the corrections to the representation of the joint distribution of $q+l$ particles, n_{q+l}, by the product $n_q n_l$ for large separation between the sets of q and l particles. For a system in which there exists a "finite correlation length," we find explicitly the $1/N$ correction term to the simple product, where N is the number of particles in our system. When $q+l$ is equal to two, this expression reduces to that familiar from the Ornstein-Zernike relations for scattering of light from a fluid. In a uniform gas, our derivation also yields the complete $1/N$ dependence of equilibrium distributions. Our result on the asymptotic form is then used to determine the low-order distribution functions for an equilibrium system of varying density, as well as for a nonequilibrium system represented by a local-equilibrium ensemble. These distribution functions are shown to be governed by the temperature and density in the vicinity of the molecules considered. We find, as expected, that the two-body distribution function coincides, to within quadratic terms in the gradients, with its equilibrium value for a uniform system at the temperature and density of the midpoint. For the higher order distributions, correction terms linear in the gradients are found.

PLASMA DENSITY FLUCTUATIONS IN A MAGNETIC FIELD. E. E. Salpeter,* Laboratory of Nuclear Studies and Center for Radiophysics and Space Research, Cornell University, Ithaca, New York and School of Physics, Sydney University, Sydney, N.S.W., Australia (Received February 10, 1961).

Sinusoidal electron charge density fluctuations with propagation vector $\vec{k}$ are considered for a

fully ionized gas in complete thermodynamic equilibrium in a constant magnetic field. Let α and ϵ be the ratio of the Debye length and of an electron gyroradius, respectively, to the wavelength k^{-1}. A general formula is derived for the frequency spectrum of these fluctuations for arbitrary values of α, ϵ, and the angle $(\phi - \frac{1}{2}\pi)$ between $\vec{k}$ and the magnetic field. The dispersion relation implied by this expression has been obtained previously by Gross and by Bernstein, but the method of derivation is different. A very small electron-ion mass ratio m/M is assumed.

For large values of ϵ and α most of the intensity occurs at small frequencies: If one has $\sin\phi \gg (m/M)^{1/2}\epsilon$, the main spectrum is continuous as in the absence of a magnetic field; if $(m/M)^{1/2} \ll \sin\phi \ll (m/M)^{1/2}\epsilon$, it consists of lines with spacing about the ion gyrofrequency; if $\sin\phi \ll (m/M)^{1/2}$, it consists mainly of a line at zero frequency. Weaker spectral lines are obtained which correspond to plasma oscillations, the existence of "frequency gaps" is confirmed for small angles ϕ, and the intensities of the various components are evaluated. For small ϕ, another spectral line is obtained at a "resonance" frequency intermediate between the electron and ion gyrofrequency.

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EFFECT OF INTERACTIONS ON DETERMINATION OF FERMI SURFACES. Edward A. Stern, Department of Physics, University of Maryland, College Park, Maryland (Received September 6, 1960).

The effect of both electron-electron and electron-phonon interactions on a degenerate electron gas in a uniform positive background is considered. It is shown that when electron-electron interactions alone are considered, the free electron mass is still measured by cyclotron resonance, the Faraday effect, and optical constants. However, the period of the de Haas–van Alphen oscillations is changed from what one calculates neglecting interactions and is changed in the same way as the specific heat. When electron-phonon interactions are added everything changes. In particular, it is shown that the cyclotron mass is no longer the free value, and the de Haas–van Alphen period and the specific heat are changed in different ways. Comparison with measurements on aluminum which approximates the model used shows that both electron-phonon and electron-electron effects are important and of the same magnitude.

EXPONENTIAL TEMPERATURE DEPENDENCE OF YOUNG'S MODULUS FOR SEVERAL OXIDES. J. B. Wachtman, Jr., W. E. Tefft, D. G. Lam, Jr., and C. S. Apstein, National Bureau of Standards, Washington, D. C. (Received February 3, 1961).

Young's modulus was measured over the temperature range 77°K to 850°K by an accurate resonance technique. Data are presented for single crystals of aluminum oxide with various orientations of the crystallographic axes and for polycrystalline aluminum oxide, thorium oxide, and magnesium oxide. The results show that the range of validity of a T^4 temperature dependence predicted by theory must be quite small. The temperature dependence is very well described over the whole temperature range by $T\exp(-T_0/T)$, where T_0 is an empirical parameter.

ELECTRON-ELECTRON SCATTERING AND TRANSPORT PHENOMENA IN NONPOLAR SEMICONDUCTORS. Joachim Appel, John Jay Hopkins Laboratory for Pure and Applied Science, General Atomic Division of General Dynamics Corporation, San Diego, California (Received November 30, 1960).

The effect of electron-electron scattering processes due to Coulomb forces on the transport phenomena in nonpolar isotropic solids is treated in the framework of Kohler's variation principle. By considering the conduction electrons as a Fermi-Dirac gas of noninteracting free quasi-particles, each with charge $-e$ and mass m^*, electron-electron scattering is taken into account as a small perturbation, as is electron-phonon scattering in nonpolar solids. A shielded Coulomb potential which depends on two parameters—the effective dielectric constant and the shielding constant—is used as the interaction potential. These two parameters, for small concentrations of electrons, may be assumed to be independent of the distance between two electrons during a scattering process.

A general qualitative result is that electron-electron scattering causes the electrical conductivity to be reduced less than the electronic heat

conductivity. The conductivities and the Wiedemann-Franz ratio will be reduced by an amount determined by the energy dependence of that perturbation of the electron distribution which is induced by primary scattering sources such as electron-phonon scattering or electron-impurity scattering. Quantitative results for nondegenerate semiconductors are obtained in terms of the variational method. With electron-phonon and electron ion scattering assumed in turn as the primary scattering mechanism, the influence of electron-electron scattering on the electrical conductivity, the heat conductivity, and the Seebeck coefficient is calculated as a function of temperature. The results are discussed with respect to the experimental situation.

The effect of electron-electron scattering on transport phenomena in metals is briefly considered. The applicability of the results obtained for isotropic semiconductors to an important class of anisotropic semiconductors is shown.

MAGNETIC COUPLING IN Pd-DILUTE IRON GROUP ALLOYS. E. O. Wollan, Oak Ridge National Laboratory, Oak Ridge, Tennessee (Received October 28, 1960; revised manuscript received March 10, 1961).

The magnetic properties of palladium and its alloys with iron group elements are discussed in terms of the splitting of the d orbitals. The paramagnetic properties of pure palladium are accounted for on the basis that the holes in the d shell are associated with the nonoverlapping e orbitals, whereas the ferromagnetic coupling in the face-centered $3d$ elements and their alloys is associated primarily with holes in the overlapping t orbitals. On this basis and on the basis of a change in the splitting when palladium is alloyed with $3d$ metals, it is possible to account for the paramagnetic and ferromagnetic properties of the Pd-dilute iron group alloys. Because of the larger amount of available data, attention is given primarily to the Pd-Fe system.

PRESSURE EFFECT ON VACANCY MIGRATION RATE IN GOLD. Roy M. Emrick,* Department of Physics, University of Illinois, Urbana, Illinois (Received January 23, 1961).

The effect of hydrostatic pressures up to 10 000 kg/cm^2 on the annealing rate of vacancies quenched in gold is studied. High-purity gold wires are quenched from 700°C, trapping in the equilibrium concentration of vacancies at that temperature. The vacancies are then observed to anneal out in the vicinity of room temperature at various pressures by observing the decay of the quench-induced residual resistance increase. A simple relation for the temperature dependence of the annealing rate is derived, assuming a random distribution of vacancies and vacancy sinks. By applying thermodynamic relations to the expression for the vacancy annealing rate, a volume of motion is derived in terms of the experimentally determined pressure effect on the annealing rate. For gold, the motional volume is found to be 1.50 ± 0.14 cm^3/mole compared with the atomic volume of 10.2 cm^3/mole. A hard-spheres model for the jump process predicts about a one-atomic-volume increase of the lattice for the saddle point configuration as well as a one-atomic-volume increase when a vacancy is formed. Many theoretical calculations of the lattice distortion around a vacancy and indirect experimental measurements of the volume change of the lattice on forming a vacancy indicate that there is considerable relaxation of the neighboring atoms about a vacancy. These results are used to explain the small value of the motional volume. Activation volumes derived from measurements of the effect of pressure on the rate of self-diffusion are seen to be consistent with the present experimental value.

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ADIABATIC DEMAGNETIZATION IN A ROTATING REFERENCE SYSTEM. C. P. Slichter and William C. Holton,* Department of Physics, University of Illinois, Urbana, Illinois (Received February 9, 1961).

Redfield has proposed that under some circumstances, a magnetic resonance should be described by saying that the spin system has achieved a temperature in a reference system which rotates at the frequency of the applied alternating field. He based his proposal on experiments in which characteristic times of observation were long compared to the spin-lattice relaxation time. A theory of spin-lattice processes was necessary to analyze the results. We describe a set of experiments to verify his hypothesis, using times short compared to the

spin-lattice relaxation, which test his hypothesis without need for a theory of spin-lattice relaxation. The experiments are shown to be similar to conventional adiabatic demagnetization, performed, however, in a rotating reference frame. Redfield's ideas are thereby presented in a particularly simple form. The differences between reversible and irreversible losses in magnetization are illustrated, and it is shown, for example, that one can invert the magnetization with respect to the static field by passing through the resonance using alternating fields much less than the linewidth. The studies were made using the Na resonance in NaCl.

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INTERNAL MAGNETIC FIELDS IN MANGANESE-TIN ALLOYS. Luise Meyer-Schützmeister, R. S. Preston, and S. S. Hanna, Argonne National Laboratory, Argonne, Illinois (Received February 13, 1961).

The hyperfine fields at the tin sites in two manganese-tin alloys have been studied as a function of temperature to above the Curie points. In addition to the Zeeman splittings, observed and analyzed previously, a possible quadrupole interaction of about 27 Mc/sec is observed in Mn_2Sn. In Mn_4Sn the hyperfine field is small and negative, about -45 koe; in Mn_2Sn it is large and positive, about +200 koe. As in the case of the pure ferromagnetic transition elements, it seems necessary to invoke a positive term associated with conduction-electron polarization and a negative one arising from core polarization to explain these results.

LOW-FREQUENCY CONDUCTIVITY DUE TO HOPPING PROCESSES IN SILICON. M. Pollak* and T. H. Geballe, Bell Telephone Laboratories, Murray Hill, New Jersey (Received February 1, 1961).

The complex conductivity has been measured in n-type silicon with various kinds of impurities at frequencies between 10^2 and 10^5 cps and temperatures between 1 and 20°K. In most cases it is orders of magnitude larger than the measured dc conductivity and is attributed to polarization caused by hopping processes. The observed frequency dependence in the measured range can be expressed as $A\omega^{0.8}$, where A is a complex constant. At the low-temperature end, the conductivity is roughly proportional to minority impurity concentration and is almost independent of the majority impurity concentration. At higher temperatures the conductivity becomes approximately proportional to the product of both concentrations. A simple theory, based on the currently accepted model of impurity conduction, is given for the higher temperature range. It accounts well for the observed frequency and concentration dependences. However, only order-of-magnitude absolute agreement is obtained.

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DYNAMIC POLARIZATION OF NUCLEI BY ELECTRON-NUCLEAR DIPOLAR COUPLING IN CRYSTALS. O. S. Leifson and C. D. Jeffries, Department of Physics, University of California, Berkeley, California (Received February 2, 1961).

This paper is concerned with a detailed investigation of the dynamic polarization of the protons in $(Ce, La)_2Mg_3(NO_3)_{12}\cdot 24H_2O$ which occurs when one saturates the "forbidden" microwave transitions that simultaneously flip a proton spin and a Ce^{3+} electron spin. The rate equations for the electron and nuclear polarization are solved for (a) a simple ideal model, (b) a model for the case where the forbidden lines are not resolved, and (c) a model taking into account nuclear spin temperature diffusion. An apparatus for simultaneous observation of proton magnetic resonance and Ce^{3+} paramagnetic resonance at liquid helium temperatures is described. The Ce^{3+} spin-lattice relaxation time T_{1e} is directly measured by a transient method and it is found that $T_{1e} \propto T^{-1.4\pm.2}$ for temperatures in the range $1.9°K < T < 2.7°K$. In the same crystals, the proton relaxation time T_{1n} is also measured by a transient method and found to be $T_{1n} \propto T^{-7}$ and dependent on the concentration of Ce^{3+} ions. The relative magnitudes of T_{1n} and T_{1e} are best explained by a model intermediate between (a) and (c). At $T \approx 1.5°K$ and a microwave frequency $\nu_e \approx 9.3$ kMc/sec, the proton polarization is observed for a number of different concentrations of Ce^{3+}. The magnitude of the polarization, its dependence on magnetic field and microwave power, and the transient behavior are studied and qualitatively explained. In a crystal containing 1% Ce the proton polarization is observed to become greater than the ther-

mal equilibrium value by the factor 150, which is about one-quarter of the theoretical ideal. At higher microwave frequencies ($\nu_e \approx 50$ kMc/sec) it should be possible to obtain in this crystal sufficient proton polarization (~25%) to be useful for dynamic nuclear cooling experiments and nuclear targets.

LOW-TEMPERATURE OPTICAL BLEACHING OF F CENTERS IN KCl. A. R. Reinberg* and L. I. Grossweiner, Department of Physics, Illinois Institute of Technology, Technology Center, Chicago, Illinois (Received February 3, 1961).

Optical bleaching of F centers in x-rayed and additively colored KCl has been studied at 77°K with pulse irradiation methods. Constant "F light" incident on the colored crystal establishes a photostationary equilibrium between F and F' centers. The application of a high-intensity light flash of appropriate spectrum displaces the system from equilibrium to either direction. The return to equilibrium is exponential in time, with a rate controlled by the intensity and spectrum of the constant light, but not depending on the F-center concentration or the method of coloration. The results are in agreement with a kinetic model involving only F-to-F' center interconversions. Prior room-temperature optical bleaching inhibits the extent and rate of low-temperature bleaching, probably due to the growth of other bands under the F band. The saturation of the extent of bleaching at high pulse-light intensity and a lower bleaching efficiency for the additively colored crystal indicate that the distribution of F centers in the crystal is significant in low-temperature bleaching.

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ELASTIC CONSTANTS OF IRON FROM 4.2 TO 300°K. J. A. Rayne and B. S. Chandrasekhar, Westinghouse Research Laboratories, Pittsburgh, Pennsylvania (Received February 1, 1961).

The zero-field elastic constants of iron have been measured from 4.2 to 300°K using the ultrasonic pulse technique. Extrapolation of the data to absolute zero gives $c_{11} = 2.431 \pm 0.008$, $c_{12} = 1.381 \pm 0.004$, and $c_{44} = 1.219 \pm 0.004$, all expressed in units of 10^{12} dyne cm^{-2}. The corresponding limiting value of the Debye temperature is $\theta_0 = 447 \pm 2$°K. Using this figure, the low-temperature heat capacity data for iron have been reanalyzed assuming the presence of a spin-wave contribution to the specific heat; i.e., the heat capacity is assumed to follow the relation $C = \gamma T + \beta T^3 + \alpha T^{3/2}$. A least-squares fit of $(C - \beta T^3)/T$ versus $T^{1/2}$ gives $\gamma = (11.7 \pm 0.1) \times 10^{-4}$ cal mole^{-1} deg^{-2}, $\alpha = (2 \pm 1) \times 10^{-5}$ cal mole^{-1} deg$^{-5/2}$. There is agreement, within experimental error, between the latter figure and the theoretical estimate of $\alpha = 0.8 \times 10^{-5}$ cal mole^{-1} deg$^{-5/2}$, obtained from the low-temperature magnetization data of Fallot. From the room-temperature elastic constants, the compressibility of iron is found to be $\beta = (5.95 \pm 0.02) \times 10^{-13}$ cm^2 dyne^{-1}, which agrees exactly with the static value obtained by Bridgman.

INFLUENCE OF A STATIC ELECTRIC FIELD ON POSITRONIUM FORMATION IN POLYMERS. A. Bisi, F. Bisi, A. Fasana, and L. Zappa, Istituto di Fisica del Politecnico, Milano, Italy (Received January 31, 1961).

The effect of a static electric field on formation of positronium in some polymeric materials was detected by a study of the time-delay spectrum of the annihilation γ rays. Typical nonpolar polymers, like polyethylene and Teflon, show a strong decrease in positronium formation with increasing electric field (33% and 13%, respectively, at 50 kv/cm), while no effect was found in polar polymers, like Lucite, nylon, and polyvinyl chloride.

ATOMIC-BEAM INVESTIGATIONS OF ELECTRONIC AND NUCLEAR GROUND STATES IN THE RARE EARTH REGION. Amado Y. Cabezas, Ingvar Lindgren, and Richard Marrus, Lawrence Radiation Laboratory and Department of Physics, University of California, Berkeley, California (Received February 9, 1961).

A number of radioactive isotopes in the rare earth region have been investigated by using the atomic-beam magnetic-resonance technique. The total electronic angular momentum (J) and the atomic g value (g_J) have been determined for some low-lying levels in Pm, Dy, Ho, and Er. These observations are consistent with the

following ground-state assignments: Pm I – $(4f)^5(6s)^2$, $^6H_{5/2}$; Dy I – $(4f)^{10}(6s)^2$, $^5I_8$; Ho I – $(4f)^{11}(6s)^2$, $^4I_{15/2}$; and Er I – $(4f)^{12}(6s)^2$, $^3H_6$. The ground-level assignments agree with the Hund's-rule predictions, and the g values approximate the Landé values well. Our present experimental knowledge concerning electronic ground states in the rare-earth region is summarized. The following nuclear spin values have been obtained: Pm^{149}, $I=7/2$; Pm^{151}, $I=5/2$; Gd^{159}, $I=3/2$; Tb^{160}, $I=3$; Dy^{165}, $I=7/2$; Dy^{166}, $I=0$; 27-hr Ho^{166}, $I=0$; Er^{169}, $I=1/2$; Er^{171}, $I=5/2$; and Tm^{171}, $I=1/2$. These values are compared with available beta-decay results. Most of these isotopes have large nuclear deformations, and the spin values are discussed in relation to the single-particle energy-level diagrams given by Nilsson. A few results are explained with the shell model.

THEORY OF ZEEMAN EFFECT IN THE GROUND MULTIPLETS OF RARE EARTH ATOMS. B. R. Judd and I. Lindgren, Lawrence Radiation Laboratory, University of California, Berkeley, California (Received February 9, 1961).

A number of corrections are made to the simple Landé formula for the g values of levels deriving from the ground term of configurations of the type $4f^n$. These include (a) the Schwinger correction, to give an accurate value of the gyromagnetic ratio for the electron spin; (b) a correction to allow for the deviations from perfect RS coupling; (c) a relativistic correction, which is directly related to the kinetic energy of the electrons; (d) a diamagnetic correction, depending on the electron density of the core. In order to calculate (b), the spin-orbit coupling constants and the Coulomb integrals F_k are estimated either from existing spectroscopic data or from a process of interpolation or extrapolation. An argument is presented for taking ratios of the integrals F_k corresponding to a hydrogenic eigenfunction. The various radial integrals required in the calculation of (c) and (d) are derived from a modified hydrogenic eigenfunction of the form $r^n e^{-ar}\cosh\kappa(ar-n)$. The parameter κ in this expression is estimated to be approximately 0.42 over the rare earth series by comparison with available Hartree self-consistent field eigenfunctions. The second parameter a is chosen to give a fit with the spin-orbit coupling constants. The result of calculating (a), (b), (c), and (d) is to give atomic g values which agree remarkably well with the experimental data. This confirms that the ground configurations of Pr I, Nd I, Pm I, Sm I, Eu I, Dy I, Ho I, Er I, and Tm I are of the type $4f^n$, and that such a configuration is very low lying in Tb I. Tables of spin-orbit coupling constants and $\langle r^{-3}\rangle$ for both neutral and triply ionized rare earth atoms are given, as well as tables of other radial integrals.

CONFIGURATION INTERACTION IN SIMPLE ATOMIC SYSTEMS. A. W. Weiss,* Laboratory of Molecular Structure and Spectra, Department of Physics, University of Chicago, Chicago, Illinois (Received February 8, 1961).

The ground-state wave functions of the helium, lithium, and beryllium atoms were approximated by a superposition of configurations with expansion lengths ranging from 35 for helium to 55 for beryllium. The discrepancies in the total energy are 0.014 ev for helium, 0.026 ev for lithium, and 0.17 ev for beryllium. A 19-configuration function was also applied to the lowest ^3S state of helium, with a resulting accuracy of 0.0005 ev. The calculations were also made on all the isoelectronic series of ions through $Z=8$, the discrepancy remaining of the same order of magnitude but increasing with increasing Z. A lower bound to the electron affinity of lithium is set at 0.4773 ev, with the most probable value, obtained by extrapolating the isoelectronic series, being placed at 0.62 ev.

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CANCELLATION OF KINETIC AND POTENTIAL ENERGY IN ATOMS, MOLECULES, AND SOLIDS. M. H. Cohen,* Institute for the Study of Metals, University of Chicago, Chicago, Illinois and Hughes Research Laboratories, Malibu, California, and V. Heine, Cavendish Laboratory, Cambridge, England (Received December 16, 1960).

In the energy levels of valence electrons in atoms, molecules, solids, and liquids, there is a contribution from the large negative potential energy inside the core of the atom and the large positive kinetic energy which the electron has

there. Phillips and Kleinman have shown how the kinetic energy can be represented by a repulsive pseudopotential which cancels most of the potential energy inside the core. The explicit representation of the pseudopotential is now developed further to demonstrate more clearly the extent of the cancellation. The formalism justifies the simple models which are in common use for treating valence electrons. It is also used to relate similar atoms from different rows of the periodic table, and in particular to discuss the systematic trends in the energy levels of the alkali and noble metal atoms.

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LOW-ENERGY ELECTRON SCATTERING FROM ATOMIC OXYGEN. R. H. Neynaber, Lawrence L. Marino, Erhard W. Rothe, and S. M. Trujillo, Physics Section, Convair, San Diego Division, San Diego, California (Received February 13, 1961).

The total cross section for the scattering of electrons by atomic oxygen has been measured as a function of electron energy from 2.3 to 11.6 ev. The number of electrons scattered from a region defined by the intersection of an electron beam and a modulated molecular oxygen beam was compared with the number scattered when the oxygen beam was partially dissociated. A radio-frequency discharge dissociated about 30% of the molecules. The degree of dissociation was measured with a mass spectrometer. From the data, the ratios of atomic to molecular scattering cross sections were obtained. The absolute atomic values were calculated by multiplying these ratios by the molecular oxygen cross sections obtained by Brüche. The result is a virtually constant cross section of $(6.2 \pm 0.5)\pi a_0^2$ in the entire energy range studied. These results are compared with five theoretical estimates.

NEW PROCESS OF EXCITATION TRANSFER IN HELIUM. Robert M. St. John and Richard G. Fowler, Department of Physics, University of Oklahoma, Norman, Oklahoma (Received August 8, 1960; revised manuscript received February 27, 1961).

Precise observations have been made on light emission from low triplet levels of helium gas traversed by a monoenergetic electron beam, with varying helium pressure and electron energy. In the past it has been customary to assign this triplet excitation to direct transfer from ^1P states in flagrant violation of the Wigner rule and seemingly excessive cross sections for the transfer have been inferred.

A new process of excitation transfer is proposed which minimizes the conflict with the Wigner rule by reducing the sizes of the cross sections required to values close to the gas-kinetic cross section. It is hypothesized that many ^1P states, including those with large quantum number n, transfer excitation energy to neighboring triplet states having closely corresponding principal quantum numbers. The triplet states thus formed in turn populate low-level triplet states by radiative transitions. It is found that states lying between $n=4$ and $n=15$ would play the dominant role in the transfer process.

Satisfying qualitative explanations of several additional excitation-transfer-process phenomena are derived from this new multiple-state transfer process.

INTERACTION OF NEUTRONS WITH He^3. A. R. Sayres, K. W. Jones,* and C. S. Wu, Columbia University, New York, New York (Received February 7, 1961).

Spectra of the $n + He^3$ reactions were obtained for monoenergetic neutron fluxes of 0.95, 2.67, 5.00, 8.07, and 17.5 Mev using a He^3-filled proportional counter. Analysis of these spectra yielded the ratios to the total cross section of the total elastic, the $He^3(n,p)H^3$, and the $He^3(n,d)D$ reaction cross sections. Absolute cross sections were obtained by normalization to the known total cross section. Differential elastic scattering cross sections for neutrons on He^3 were obtained through the relationship between the scattering angle of the neutron and the observed energy of the He^3 recoil in the counter filling. These angular distributions are compared with the theoretical angular distributions as calculated by Bransden, Robertson, and Swan. The present experiment was intended to discriminate between their assumed two special cases for the potential inter-

action: the symmetrical exchange force and the Serber exchange force. The experimental results favor the Serber exchange force. This experiment also was intended to make possible the analysis of more complex neutron spectra by providing knowledge of $n+He^3$ reaction cross sections and the spectra obtained when only monoenergetic neutrons are present. A comparison is made of the results of this experiment with cross sections calculated from inverse reactions and direct measurements of other investigators, where such data are available.

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DECAY OF I^{132}. R. L. Robinson, E. Eichler, and Noah R. Johnson, Oak Ridge National Laboratory, Oak Ridge, Tennessee (Received January 18, 1961; revised manuscript received March 17, 1961).

The decay of I^{132} has been investigated by means of scintillation spectrometers. Energies (and relative intensities) of the gamma rays which were observed are 0.240(1.3), 0.518(15), 0.667(100), 0.72(5), 0.775(63), 0.953(15), 1.14(1.2), 1.142(2.7), 1.30(2.4), 1.392(6.4), 1.45(1.1), 1.75(0.3), 1.91(0.7), 1.99(0.8), 2.08(0.18), 2.18(0.13), and 2.39(0.11) Mev. Our data indicate that the gamma-ray peak at 0.667 Mev actually consists of four gamma rays with energies between 0.62 and 0.68 Mev. An energy level diagram of Xe^{132} based on the present spectral studies and gamma-gamma angular correlation measurements has been proposed. Energies (and spins) of the levels are 0.673(2+), 1.32, 1.448(4+), 1.81, 1.966(3), 2.10(3 or 4), 2.401(4), 2.59(3), and 2.84(3, 4, or 5) Mev.

ELASTIC SCATTERING OF HEAVY IONS FROM SEVERAL GAS TARGETS. E. Newman,* P. G. Roll,† and F. E. Steigert, Yale University, New Haven, Connecticut (Received February 13, 1961).

The absolute differential cross section for the elastic scattering of oxygen by gaseous targets of helium, nitrogen, and neon and of carbon by nitrogen were measured from about 4 to ~ 20 degrees in the center-of-mass system. The energy of the incident heavy ions was ~ 10 Mev per atomic mass unit. The ratio of the elastic cross section to the Coulomb cross section exhibits diffraction-like oscillations which damp out as $\eta = Z_1 Z_2 e^2/\hbar v$ increases from ~1 to ~4. The results are compared with the predictions of a simple diffraction model, a sharp-cutoff model, and a modified-cutoff model. Reasonably good agreement is obtained in predicting the angular position of the maxima and minima, but not the absolute cross section. The modified-cutoff model appears to apply only if η is greater than about 2.5. The value of r_0 found for the O^{16}-Ne scattering is 25% greater than for the remaining three reactions.

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LOW-ENERGY CONVERSION ELECTRONS OF Ag^{106} AND Rh^{106}; Pd^{106} LEVELS. W. G. Smith, Physics Department, Purdue University, Lafayette, Indiana (Received January 27, 1961).

The low-energy (less than 0.935 Mev) conversion electrons emitted in the decays of Ag^{106} and Rh^{106} have been observed in two permanent-magnet electron spectrographs. A total of 29 transitions were observed in the Ag^{106} decay; 2 transitions were observed in the Rh^{106} decay. These data and the gamma scintillation results of Robinson et al. on the same decays were used to postulate the following levels in Pd^{106}: 0, (0+); 0.5116, (2+); 1.1272, (2+); 1.1331, (0+); 1.2287, (4+); 1.5568, (3, 4+); 1.7020, (2+); 1.9310, (3, 4+); 2.0825, (3+); 2.3040, (3, 4+); 2.3489, (3, 4+); 2.3636 or 1.9469, (3, 4, 5+); 2.7336, (5, 6+); 2.7540, (5+); and 2.9494 Mev, (5, 6+). The proposed level scheme indicates that ~5%, ~85%, and ~10% of the electron capture of Ag^{106} proceeds to the 2.9494-, 2.7540-, and 2.7336-Mev levels, respectively.

EXCITATION FUNCTIONS OF THE $(p, 2p6n)$ AND $(p, 3p5n)$ REACTIONS FOR 60-, 100-, 150-, AND 240-Mev PROTONS ON ENRICHED ZIRCONIUM-90. Envare Unseren* and Edwin O. Wiig, University of Rochester, Rochester, New York (Received August 31, 1959; revised manuscript received March 17, 1961).

The cross sections of the $Zr^{90}(p, 3p5n)Sr^{83}$ and $Zr^{90}(p, 2p6n)Y^{83}$ reactions have been determined for proton energies of 60-240 Mev. The cross

section for even-odd Sr^{83} production in the energy range 100-240 Mev is 1330-2140 times as large as that for odd-even Y^{83}, as contrasted with much smaller values for the production of odd-odd and even-even isobars in similar reactions.

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HALF-LIFE OF B^{12}, Na^{24m}, AND As^{75m}. Alois W. Schardt, Los Alamos Scientific Laboratory, University of California, Los Alamos, New Mexico (Received February 7, 1961).

The following half-lives were measured: B^{12}, $(20.31 \pm 0.20) \times 10^{-3}$ sec; the 472-kev level of Na^{24}, $(19.9 \pm 0.3) \times 10^{-3}$ sec; and the 305-kev level of As^{75}, $(16.8 \pm 0.4) \times 10^{-3}$ sec. The activities were made with a pulsed beam from a Van de Graaff generator. The data were taken with gated radiation-detection circuits in conjunction with a 9-channel time-delay analyzer, and the half-lives were determined by least-squares analysis.

ABSOLUTE CROSS SECTIONS OF THE REACTIONS $Na^{23}(n,p)Ne^{23}$ AND $Na^{23}(n,\alpha)F^{20}$. Claude F. Williamson,* Department of Physics, The University of Texas, Austin, Texas (Received February 3, 1961).

The absolute cross sections for the competing reactions $Na^{23}(n,p)Ne^{23}$ and $Na^{23}(n,\alpha)F^{20}$ have been measured over the energy range 4 to 19 Mev by the activation method using neutrons from the $H^2(d,n)He^3$, $N^{14}(d,n)O^{15}$, $C^{14}(d,n)N^{15}$, and $H^3(d,n)He^4$ reactions. The excitation curves thus obtained are compared with theoretical results based upon the statistical theory. A fair fit is obtained for the (n,p) reaction, but serious discrepancies exist between the measured and calculated (n,α) cross sections.

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SMALL-ORDER SHAPE FACTOR IN Na^{22}. R. T. Nichols,* R. E. McAdams, and E. N. Jensen, Institute for Atomic Research and Department of Physics, Iowa State University, Ames, Iowa (Received February 15, 1961).

The beta spectrum of Na^{22} has been examined using an intermediate-image beta-ray spectrometer operating at $2\frac{1}{2}\%$ transmission and 2.4% resolution. Five separate determinations of the maximum energy and of the slope of the form-factor plot were made. One determination was made using a charge-discriminating baffle inserted in the spectrometer. The unweighted mean of the five values for the maximum energy is 547.4 ± 1.0 kev. The unweighted mean of the five values for the form-factor slope is $(0.023 \pm 0.006)/mc^2$.

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EXCITATION FUNCTIONS OF THE (p,pn) AND $(p,2p)$ REACTIONS ON Ce^{142} AT 60-233 Mev. William R. Ware* and Edwin O. Wiig, Department of Chemistry, University of Rochester, Rochester, New York (Received September 8, 1960; revised manuscript received March 13, 1961).

The cross sections of the reactions $Ce^{142}(p,pn)Ce^{141}$ and $Ce^{142}(p,2p)La^{141}$ have been determined for protons of 60-, 120-, 180-, 200-, and 233-Mev energy. A comparison is made of the observed yields with those predicted on the basis of Monte Carlo cascade-evaporation calculations at 83, 238, and 368 Mev. For protons of energy between 60 and 240 Mev the agreement is good for the (p,pn) reaction and fair for the $(p,2p)$.

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DECAY SCHEME OF Sn^{113}. R. C. Greenwood* and E. Brannen, University of Western Ontario, London, Ontario, Canada (Received February 7, 1961).

The electron capture decay of Sn^{113} has been investigated using scintillation detectors and coincidence circuitry. Decay was found to occur to the 650-kev and 393-kev states of In^{113} with branching ratios of $(1.8 \pm 0.1)\%$ and $(98.2 \pm 0.1)\%$, respectively. The 650-kev state decayed by the emission of a 255-kev gamma ray to the 393-kev state. No evidence was found for the existence of a 648-kev crossover transition. An upper limit for the intensity of this transition was placed as 0.1% of that of the 255-kev transition. The orbital electron capture ratio to the 648-kev state was determined to be $P_{LMN\ldots}/P_K < 0.19$.

From this value, a minimum decay energy to the 648-kev state was assigned as 150 kev. It was determined that transitions to the 393-kev state of In^{113} were first forbidden by considering logft values; which is consistent with a spin and parity of $\frac{1}{2}+$ or $\frac{3}{2}+$ for the ground state of Sn^{113}. The 648-kev state was assigned a spin and parity of $\frac{1}{2}-$ or $\frac{3}{2}-$ on the basis of possible logft values and the absence of the 648-kev crossover transition.

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ANGULAR DISTRIBUTION OF SHOWER PARTICLES FROM 1000-Bev/NUCLEON ALPHA PARTICLES ON EMULSION NUCLEI. P. L. Jain, Department of Physics, University of Buffalo, Buffalo, New York (Received January 10, 1961; revised manuscript received March 1, 1961).

Twenty-eight interactions of α particles were located in a 22-liter stack of nuclear emulsion by tracing back showers of minimum-ionization particles to their origins. The angular distributions of seventeen α particles with a dip angle $\leq 20°$ are presented. The inelasticity for these seventeen interactions shows large fluctuations for individual events and its mean value is 30%. The angular distributions of these α particles have been transferred into a system in which they are roughly symmetric. The degree of anisotropy of the angular distributions is in disagreement with a hydrodynamical model of nucleon-nucleus collisions. The detailed analysis of the angular distribution of composite stars for events with a high degree of anisotropy of secondaries in the center-of-mass system shows that the shape of the angular distribution is in agreement with the predictions of the "two-fireball" model of multiple meson production, both for nucleon-nucleon and for nucleon-nucleus collisions.

INVESTIGATION OF BREMSSTRAHLUNG AND PAIR PRODUCTION AT ENERGIES $>10^{11}$ ev. E. Lohrmann, Department of Physics, University of Chicago, Chicago, Illinois (Received February 3, 1961).

Pair production and bremsstrahlung at energies >100 Bev are investigated in nuclear emulsions by studying 91 primary electron-positron pairs starting high-energy cascades and the first secondary pair. The average energy of the showers is 320 Bev. The experimental results on the total number and the energy spectrum of photons radiated by electrons $\geq$100 Bev show a lack of soft photons, which is in disagreement with the Bethe-Heitler theory. The experiment agrees well with the new theory given by Landau, Pomeranchuk, Migdal, and Ter-Mikaeljan. A method for obtaining the mean free path L for direct pair production is presented which avoids the use of a correction for spurious tridents. A value $L = 12^{+7}_{-4}$ cm is obtained for an average electron energy of about 160 Bev. The conversion length of photons of average energy 320 Bev in nuclear emulsion is 34 ± 5 mm, in agreement with the theoretical value of 37 mm. The distribution of the separation between the electron and the positron of the original high-energy pair is also in agreement with the theoretical distribution. This indicates that no appreciable discrepancy can exist between experiment and the theoretical cross section for the energy partition between an electron and a positron, and the probability of large energy losses by radiation. Several high-energy showers presumably produced by μ mesons and one possible case of a double pair production are described.

POLARIZATION OF COSMIC-RAY MUONS AT SEA LEVEL. C. Scott Johnson,* Department of Physics, Washington University, St. Louis, Missouri (Received February 10, 1961).

Cosmic-ray muons at sea level are known to be partially longitudinally polarized. The object of this experiment was to measure the positive muon polarization by stopping the muons in a copper absorber and observing the up:down asymmetry in the decay electrons. Measurements were also made with iron and sulfur as control absorbers to test the symmetry of the apparatus. Since this experiment is performed in zero magnetic field, it has been possible to verify that unmagnetized iron depolarizes positive muons by precessing their magnetic moments in the strong, randomly oriented magnetic fields of the domains. The up:down ratio of decay electrons was 1.14 ± 0.02 for copper, 0.98 ± 0.02 for iron, and 1.02 ± 0.03 for sulfur. On the basis of the two-component neutrino theory, the muon polarization is found to be $\geq (21 \pm 3)\%$, where the inequality is due to

depolarization effects. On the basis of cosmic-ray data on the muon energy spectrum as a function of altitude, the polarization is expected to be about 30%. The data of the present experiment suggest that in the absence of external magnetic fields copper depolarizes positive muons, as there appears to be a decrease in the up:down ratio with time lived before decay.

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RADIATIVE CORRECTIONS TO ELECTRON-PROTON SCATTERING. Yung-Su Tsai, Institute of Theoretical Physics, Department of Physics, Stanford University, Stanford, California (Received November 15, 1960).

The radiative corrections to the electron-proton scattering are calculated with the effects of the proton recoil taken into account. We assumed the experimental conditions of Hofstadter et al. at Stanford; namely, only the final electrons are momentum-analyzed. The anisotropy in the maximum energy of photons which can be emitted and the radiation from the proton current are the two main effects due to the proton recoil, and both effects are considered. The mesonic effects in the two-photon exchange diagrams are not considered. Other than the uncertainty in the mesonic effects, our formula is good up to about 5 Bev.

TEST OF ISOTOPIC SPIN CONSERVATION FROM AN EXPERIMENTAL LIMIT ON $\sigma(d+d\rightarrow He^4+\pi^0)$. John A. Poirier and Morris Pripstein, Lawrence Radiation Laboratory, University of California, Berkeley, California (Received February 8, 1961).

The reaction $d+d\rightarrow He^4+\pi^0$, which is forbidden by isotopic spin conservation, has been looked for at the 184-inch cyclotron. A beam of deuterons of 460-Mev kinetic energy was scattered from an extended target filled with deuterium gas at 25-atm pressure and at liquid nitrogen temperature. The differential cross section in the center-of-mass system at $\theta_{c.m.}=90$ deg was measured and a value of $(18\pm2.3)\times10^{-34}$ cm^2/steradian was obtained. We consider this value to be an upper limit for the reaction; the data are consistent with no π^0 production. From a comparison of this upper limit with the theoretical prediction of the cross section for this reaction if isotopic spin need not be conserved, we conclude that isotopic spin is at least 93.5% conserved.

THEORY OF CONSERVATION OF ISOTOPIC SPIN FROM $d+d$ REACTIONS. K. R. Greider, University of California, La Jolla, California (Received February 8, 1961).

The theoretical cross section for the reaction $d+d\rightarrow He^4+\pi^0$ has been calculated in the impulse approximation. It is assumed that the matrix element contains no isospin dependence which, due to conservation laws, would ordinarily prohibit pion production. The comparison of the theoretical and experimental cross sections for the reaction yields an upper limit of 6.5% for the amount of isospin nonconservation in strong interactions. A discussion of the effects of different He^4 wave functions in the calculation is also included.

SPALLATION OF INTERSTELLAR MATTER: COSMIC-RAY INTENSITY IN THE PAST. S. N. Milford, Department of Physics, St. John's University, Jamaica, New York, and S. P. Shen, Department of Physics, St. John's University, Jamaica, New York and State University of New York, Albany, New York (Received February 2, 1961).

It is pointed out that spallation of interstellar matter by cosmic rays occurs, and that the lithium, beryllium, and boron thus produced can be used as rough indices of the average interstellar cosmic-ray intensity. Using available data, this method gives an upper limit of the order of 10 cm^{-2} sec^{-1} for the intensity of interstellar cosmic rays of kinetic energy $> \sim 50$ Mev, averaged temporally over the past few billion years and spatially over several cubic kiloparsecs in the solar neighborhood.

INVALIDITY OF THE RAPHAEL ANALYSIS FOR MODERATE-ENERGY NUCLEON-NUCLEON SCATTERING. Peter Signell and Richard Yoder, Physics Department, The Pennsylvania State University, University Park, Pennsylvania (Received February 10, 1961).

Raphael's modified effective-range expansion for nucleon-nucleon scattering is usually truncated at the second term. It is explicitly shown that the resulting series is not a good representation of the predictions of reasonable potential models for moderate-energy nucleon-nucleon scattering. In particular, it can lead to the wrong sign for the shape parameter.

PERTURBATIVE TREATMENT OF PAIRING FORCES IN MANY-FERMION SYSTEMS. M. Bolsterli, School of Physics, University of Minnesota, Minneapolis, Minnesota (Received September 6, 1960; revised manuscript received March 16, 1961).

A noncanonical transformation which allows perturbative techniques to be applied to the pairing force problem is introduced. The lowest order eigenvalue equation gives the standard results for both strong and weak coupling.

FERMI-THOMAS TYPE APPROXIMATION FOR NUCLEI. Kailash Kumar,* Institute of Advanced Studies, Australian National University, Canberra, Australia, and Rajat K. Bhaduri, Tata Institute of Fundamental Research, Bombay, India (Received January 16, 1961; revised manuscript received March 10, 1961).

The Hartree-Fock to Fermi-Thomas reduction is carried through for finite nuclei, starting with the K-matrix formulation. The resulting expression represents the nuclear energy to good accuracy in terms of the density and its first derivatives only; it differs in detail from the semi-empirical expressions previously proposed for this purpose. This improved expression shows the inadequacy of the "semi-infinite" approximation used often in earlier studies.

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FINITE PERTURBATION THEORY IN QUANTUM ELECTRODYNAMICS. I. Bialynicki-Birula,* Department of Physics and Astronomy, University of Rochester, Rochester, New York (Received January 16, 1961).

It is proved that no infinities appear in the power series expansion of the S matrix in quantum electrodynamics if one uses an improved perturbation procedure which is based on the following property of all renormalizable field theories. The dependence of solutions on the coupling constant has a singular part, nonanalytic at $g=0$. This singular dependence must be treated exactly, whereas the remaining, nonsingular, dependence can be expanded into a power series. This power series coincides with the standard renormalized expansion. All renormalization constants in every order remain finite, provided their singular dependence on the coupling constant is treated exactly. The problem of convergence of the whole series has not been investigated.

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FORMAL THEORY OF REARRANGEMENT COLLISIONS. Marvin H. Mittleman, Lawrence Radiation Laboratory, University of California, Livermore, California (Received February 13, 1961).

A new formulation of the transition amplitude for the general rearrangement collision is presented. Optical potential ideas are used in the derivation but the final result contains no mention of the optical potential. The result is a matrix element with transitions only between mutually orthogonal states.

SPIN-ORBIT INTERACTION IN TWO-PION EXCHANGE NUCLEAR POTENTIAL. Suraj N. Gupta, Department of Physics, Wayne State University, Detroit, Michigan (Received February 15, 1961).

The spin-orbit interaction term in the two-pion exchange nuclear potential is derived. It is found that while the spin-orbit force is quite small in the triplet odd states, it is large and repulsive in the triplet even states. The relationship of the pion-theoretical result with the phenomenological spin-orbit interaction is discussed.

CLASSICAL INTERPRETATION OF QUANTUM MECHANICS. James Paul Wesley, Lawrence Radiation Laboratory, University of California, Livermore, California (Received March 3, 1960; revised manuscript received January 27, 1961).

Following de Broglie, Bohm, and others, it is assumed that quantum mechanics may be interpreted causally and that the ψ function plays the role of a generating function for particle trajectories. By arguing that the ψ function should not be interpreted as a probability amplitude, a new method for generating particle trajectories is postulated. The four-momentum of a scalar particle is assumed to be given as the gradient of an unspecified function $F(\psi)$, where ψ is a pure real solution of the Klein-Gordon equation. Since the location of a particle is determined solely by

its trajectory, the probability distribution differs from $\psi\psi^*$; therefore, ordinary experimental results differing from the traditional theory may, in principle, be predicted. Particle motion and trajectories are discussed for three examples: a free particle, a particle in a box, and the double slit.

SOME CONSIDERATIONS ON GLOBAL SYMMETRY. T. D. Lee and C. N. Yang, Institute for Advanced Study, Princeton, New Jersey (Received February 3, 1961).

If the recently discovered Y^* state is related to the $T=\frac{3}{2}$, $J=\frac{3}{2}$ resonance in πp scattering, global symmetry considerations should become relevant. In this paper, global symmetry is discussed with a view of understanding its group structure. Also discussed is a possibility of reconciling the conflict, pointed out by Pais, between certain experimental results and global symmetry. The partial widths of the Y^* state are calculated and also those of the companion excited states Z^* and Ξ^*. A generalization of the quantum number G is discussed.

PRODUCTION OF PION PAIRS—ISOSPIN ANALYSIS. P. Carruthers, Laboratory of Nuclear Studies, Cornell University, Ithaca, New York (Received January 11, 1961).

The reactions $\gamma+N\rightarrow 2\pi+N$ and $\pi+N\rightarrow 2\pi+N$ are analyzed in terms of the relevant isospin amplitudes. An experiment is suggested to measure the phase difference between even and odd π-π isospin states, with a view toward detecting a resonance in the π-π system. The experimental determination of the magnitude and phase of the amplitudes for the second reaction is discussed.